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PERFORMANCE FACTORS OF A PERIODIC-FLOW HEAT EXCHANGER

Tomme J. Lambertson

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OF A
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Tomme J. Lambertson

PERFORMANCE FACTORS
OF A
PERIODIC-FLOW HEAT EXCHANGER

by
Tomme J. Lambertson
Lieutenant, United States Navy

Submitted in partial fulfillment of
the requirements of the degree of

MASTER OF SCIENCE
IN
MECHANICAL ENGINEERING

United States Naval Postgraduate School
Monterey, California

1957

PERIODIC-FLOW HEAT EXCHANGER
OF A
Tomme J. Lambertson
by
his work is accepted as fulfill-
thesis requirements for the de-
MASTER OF SCIENCE

from the
United States Naval Postgraduate School

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ABSTRACT

The differential equations and boundary conditions describing the periodic-flow heat exchanger (frequently referred to as rotary regenerator) are sufficiently complicated to preclude complete analytical solution. Because of the interest in this type of heat exchanger considerable effort has been expended to obtain useful solutions to the analytical problem of predicting its performance. Several approximate solutions have been proposed but each of these is limited due to the restricted ranges of parameters in which they apply.

A numerical finite-difference method of calculating the spatial temperature distribution in the periodic-flow type heat exchanger is presented by considering the metal "stream" in cross-flow with each of the fluid streams as two separate but dependent exchangers. In the development no assumptions are made which would restrict the range of parameters in which the analysis would be applicable. The exchanger effectiveness has been evaluated over the following ranges of dimensionless parameters:

$$1.0 \geq C_{\min}/C_{\max} \geq 0.10$$

$$1.0 \leq C_r/C_{\min} \leq \infty$$

$$1.0 \leq NTU_0 \leq 10$$

$$1.0 \geq (hA)^* \geq 0.25$$

The National Cash Register 102A general purpose digital computer was used to accommodate the large number of subdivisions necessary for accuracy and an extrapolation of data to zero element area (infinite subdivision) was used to obtain values of effectiveness good to four significant figures.

The author expresses his appreciation to Assistant Professor C.P. Howard for his direction and encouragement in this work.

TABLE OF CONTENTS

SECTION	TITLE	PAGE
1.	Introduction	1
2.	Method	4
3.	Comparison of Results	15
4.	Limitations	17
5.	Summary and Conclusions	19
6.	Tabular Results	20
7.	Graphical Results	30
8.	Bibliography	40
9.	Appendix 1 - Derivation of Iterative Equations	42
10.	Appendix 2 - Computer Program Details	50

LIST OF ILLUSTRATIONS AND TABLES

Figure		Page
1	Illustrative Matrix Arrangement and Fluid Flow	5
2	Schematic Heat Exchange Element	6
3	Schematic Composite Heat Exchanger	9
3a	Effectiveness versus Element Area	13
4-11	Periodic-Flow Heat Exchanger Effectiveness	30
12	Variation of Effectiveness with C_{\min}/C_{\max}	38
13	$\Delta E/E_{\infty}$ versus C_r/C_{\min}	39
14	Computer Program Flow Diagram	55
Table		
1-9	Periodic-Flow Heat Exchanger Effectiveness	20
10	Computer Program Commands	56

NOMENCLATURE

SYMBOLS

A = heat transfer area on side designated by subscript, sq ft

C = heat capacity rate (W_c) of fluids or rotor matrix according to subscript, Btu/hr, deg F

c = specific heat of fluids (constant pressure) or rotor matrix material depending on subscript, Btu/lb, deg F

h = unit conductance for thermal convection heat transfer, Btu/hr, sq ft, deg F

k = unit thermal conductivity, Btu/hr, sq ft, deg F/ft

K = a constant, defined as used and distinguished by numerical subscript

N = number of subdivisions of fluid or matrix solid flow "stream" according to subscript

Q = heat transfer rate, Btu/hr

T = temperature of fluid or matrix, inlet or outlet, depending on subscripts, deg F

W = mass flow rate of fluid (lb/hr) or matrix (rev/hr)(mass) according to subscript

DIMENSIONLESS PARAMETERS

E = exchanger heat transfer effectiveness; ratio of actual to thermodynamically-limited maximum possible heat transfer rate

C_{\min}/C_{\max} = capacity rate ratio of fluid flow streams

C_r/C_{\min} = capacity rate ratio of rotor matrix to minimum fluid capacity rate

$(hA)' = (hA)_n/(hA)_x$, geometric symmetry and heat transfer balance of exchanger

$NTU = (hA)/C$, number of transfer units on side designated by subscript

$$NTU_o = \frac{1}{C_{min}} \left[\frac{1}{1/(hA)_c + 1/(hA)_h} \right] = NTU_n \left[\frac{1}{1 + (hA)'} \right] ,$$

over-all number of transfer units

SUBSCRIPTS

avg = average

r = rotor matrix

i = inlet

o = outlet, or over-all in case of NTU_o

min = minimum magnitude

max = maximum magnitude

n = value on side of C_{min}

x = value on side of C_{max}

c = cold side

h = hot side

∞ = subscript on E to indicate value when $C_r/C_{min} = \infty$

1. Introduction

In the past decade considerable interest has been shown in the application of the periodic-flow heat exchanger (rotary regenerator) to the Gas-turbine cycle for exhaust-gas thermal-energy regeneration. Regeneration by whatever method applied to the gas-turbine cycle obtains, among other improvements, higher thermal efficiency, lower specific fuel consumption, improved part load performance and lower optimum pressure ratio. Particular advantages to be expected from the application of the periodic-flow heat exchanger to the gas-turbine process are discussed by Harper and Rohsenow (1)¹ and Coppage and London (2), among others. Principal among these is the possibility of a more compact unit than is possible with the conventional stationary surface types. This is due to the fact that the periodic-flow heat exchanger matrix can be finely divided, without structural difficulty, resulting in an appreciably reduced heat exchanger volume relative to a stationary type for a given effectiveness and pressure drop. The major disadvantage of the rotating periodic-flow type is the sealing difficulties encountered and the attendant leakage involved when an appreciable pressure ratio is used. This has been reduced to acceptable limits in at least two current development programs (3,4) and further refinements should follow.

Because of the interest in this type of heat exchanger there has been considerable effort expended to obtain useful solutions to the analytical problem of predicting its performance. The differential equations and boundary conditions describing the system are sufficiently complicated to preclude complete analytical solution. An analytical

¹Numbers in parentheses refer to the Bibliography at the end of the thesis.

solution is available for the special case of infinite matrix rotative speed where the expression for effectiveness takes the same form as that of a counter-flow direct type (2) and is given by

$$E_{\infty} = \frac{1 - e^{-NTU_0(1 - C_{\min}/C_{\max})}}{1 - C_{\min}/C_{\max} e^{-NTU_0(1 - C_{\min}/C_{\max})}} \quad [1]$$

and for the further condition of $C_{\min}/C_{\max} = \text{unity}$, reduces to

$$E_{\infty} = \frac{NTU_0}{1 + NTU_0} \quad [1a]$$

Consequently, several approximate methods of solution have been proposed. Perhaps the oldest is that of Hausen (6), described and elaborated by Johnson (8,9), which begins with the known solution for the initial cooling of a matrix element, initially at uniform temperature, by the passage of a cold fluid. By dividing the regenerator tube into a large number of strips of equal length and applying this elemental solution to each strip in turn, a family of simultaneous linear algebraic equations may be derived whose solution gives the desired temperature variation in the periodic case.

A second method, due to Iliffe (10), uses the solution to the problem of the initial cooling of a matrix element at arbitrary initial temperature distribution, derived by Nusselt. Using this elemental solution, the method of Iliffe leads to two simultaneous integral equations which require numerical methods for their solution. In a third method, Saunders and Smoleniec (7) replace the governing partial differential equations by their equivalent finite difference equations and effect an approximate solution by relaxational techniques. In a fourth approach, Tipler (16) replaces the thermal problem by its electrical analogue and observes the solution on an oscilloscope. In yet a fifth approach, the perturbation

method used by Schultz (8) has been extended by Jones and Fax (5) leading to algebraic statements for the departure of the regenerator effectiveness from its asymptotic value, given by equation [1], for several types of asymmetry. Each of these solutions is limited in one respect or another, primarily due to the restricted ranges of parameters in which they apply.

It is the purpose of this thesis to present the results of a finite-difference numerical analysis which was carried out with the aid of a digital computer such that there were no limitations imposed on the four independent dimensionless parameters selected.

2. Method

The conventional idealizations and boundary conditions assumed in the usual derivation of the governing differential equations (2) are as follows:

- a. The thermal conductivity of the matrix is zero in the direction of fluid flow and matrix metal "flow". It is infinite in the other direction normal to the fluid flow.
- b. The specific heats of the two fluids and matrix material are constant with temperature.
- c. No leakage of the fluids occurs either due to direct leakage or carry over, and each fluid flow is unmixed.
- d. The convective conductances between the fluids and the matrix are constant with flow length.
- e. The fluids pass in counterflow directions.
- f. Entering-fluid temperatures are uniform over the flow inlet cross section and constant with time.
- g. Regular periodic conditions are established for all matrix elements, i.e., steady state condition.

With these idealizations a typical element, Fig. 1, from each fluid side of the exchanger can be represented schematically as shown in Fig.

2. Although Fig. 1 is a disk (axial-flow) type exchanger, typical elements of a drum (radial-flow) type can be selected, with the same assumptions, which can be represented schematically in the same manner as Fig. 2.

It is apparent from Fig. 2 that each of these elements can be regarded as a crossflow exchanger with a gas stream and a metal "stream". This extension of the method of calculating crossflow exchangers to the rotary

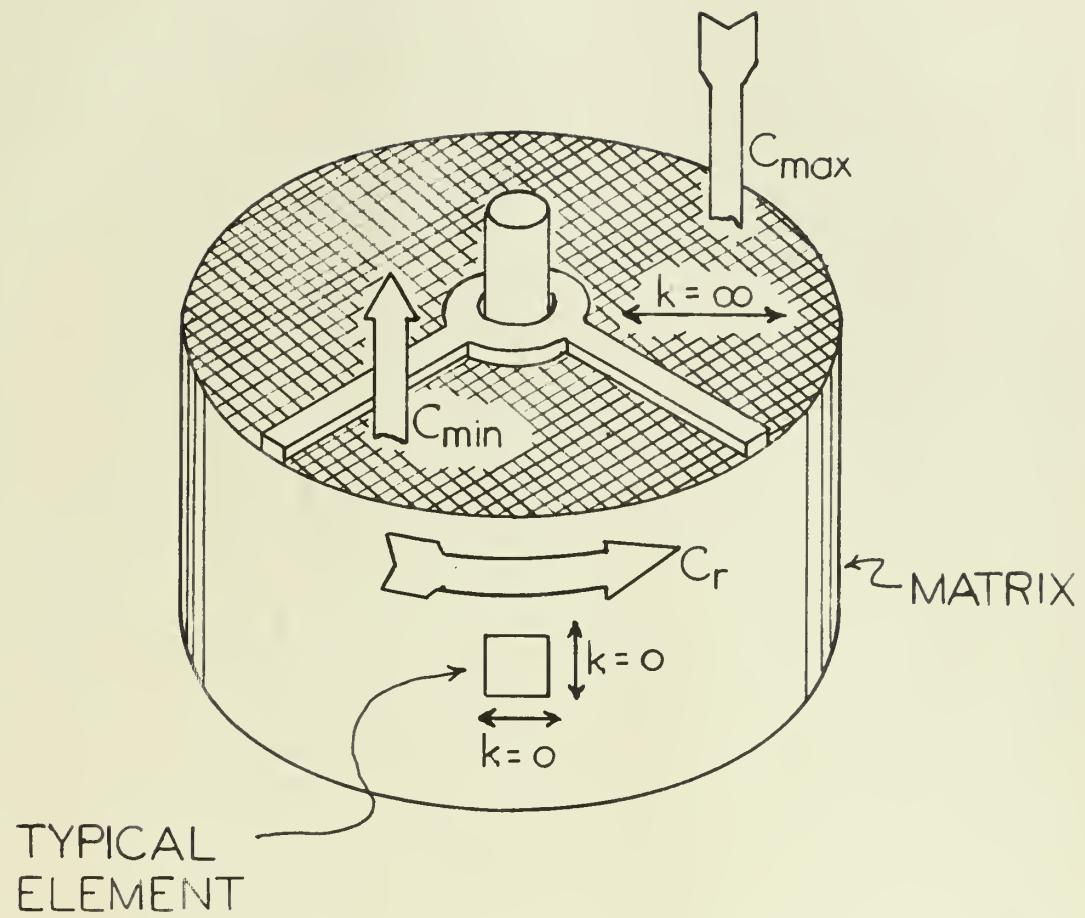
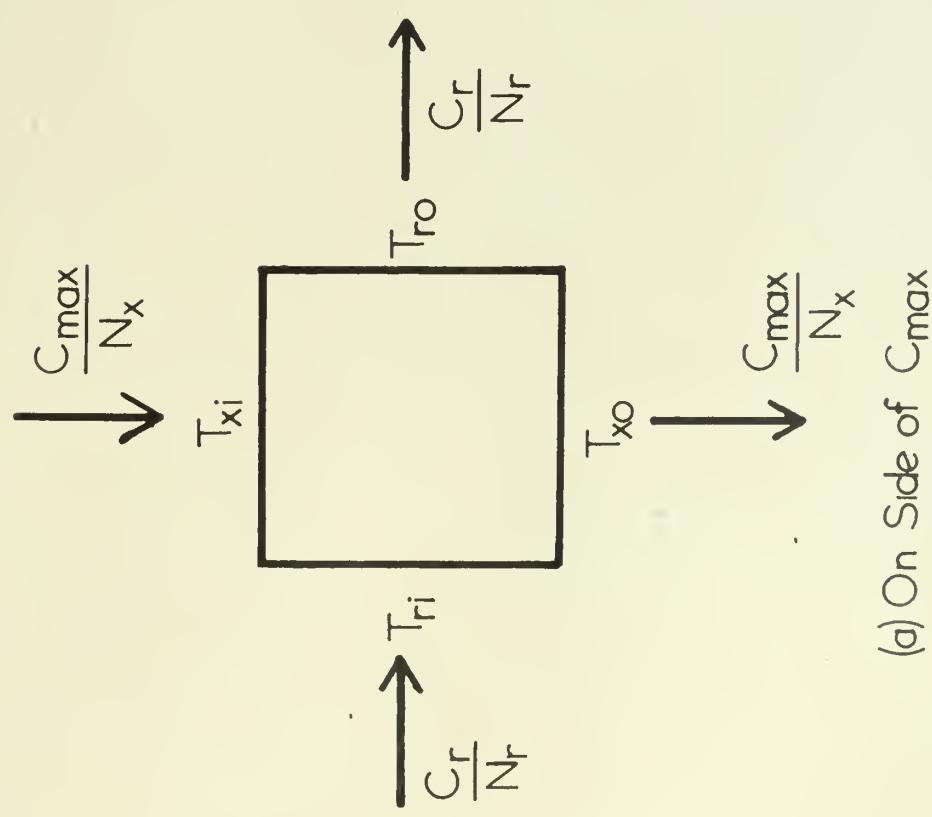
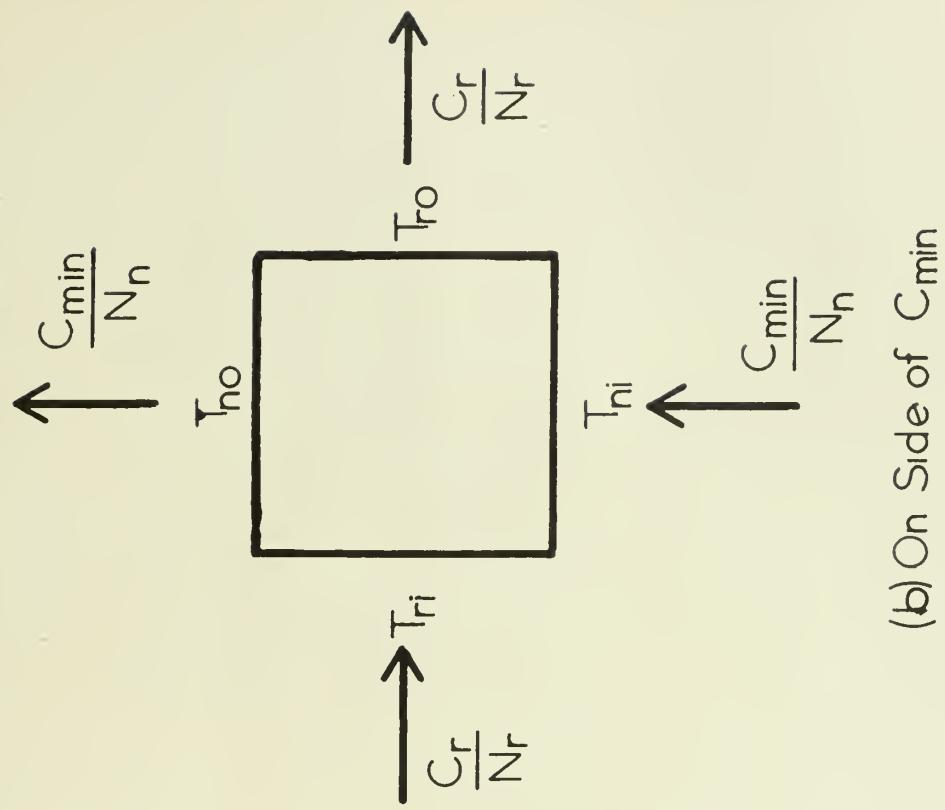


FIG. 1 ILLUSTRATIVE MATRIX
ARRANGEMENT AND
FLUID FLOW



(a) On Side of C_{\max}



(b) On Side of C_{\min}

FIG. 2 SCHEMATIC HEAT EXCHANGE ELEMENTS

regenerator was suggested by G. M. Dusinberre in his discussion of reference (2). It should be noted that the T_{ri} and T_{ro} on the two elements are not the same, but to avoid further subscript notation the side in question will be specified when it is not obvious. However, the T_{ro} of a particular element is the T_{ri} of the next adjacent element in the direction of matrix flow, and this is of particular significance at the seals where the direction of fluid flow reverses.

Consider for the moment only the heat exchange element on the side of C_{\max} , Fig 2(a). For simple unmixed cross flow with uniform temperatures across the inlets and average temperatures at the outlets, the heat transfer rate for this element is

$$Q = C_{\max}(T_{xi} - T_{xo})(1/N_x) \quad \dots \dots \dots \quad [2]$$

$$Q = C_r(T_{ro} - T_{ri})(1/N_r) \quad \dots \dots \dots \quad [3]$$

$$Q = (hA)_x \Delta T_{avg}(1/N_r N_x) \quad \dots \dots \dots \quad [4]$$

For a small enough element the arithmetic-mean temperature difference may be assumed valid, so that

$$\Delta T_{avg} = (1/2)(T_{xi} + T_{xo}) - (1/2)(T_{ri} + T_{ro}) \quad \dots \dots \quad [5]$$

By using equations [2], [3], [4], and [5] the outlet temperatures of the two streams may be solved for in terms of the inlet temperatures and put in the form (Appendix 1)

$$T_{xo} = T_{xi} - K_1(T_{xi} - T_{ri}) \quad \dots \dots \dots \quad [6a]$$

$$T_{ro} = T_{ri} + K_2(T_{xi} - T_{ri}) \quad \dots \dots \dots \quad [6b]$$

By the same method the outlet temperatures of the element on the side of C_{\min} may be solved for in terms of its inlet temperatures giving

$$T_{no} = T_{ni} + K_3(T_{ri} - T_{ni}) \quad \dots \dots \dots \quad [7a]$$

$$T_{ro} = T_{ri} - K_4(T_{ri} - T_{ni}) \quad \dots \dots \dots \quad [7b]$$

By using the definitions of the parameters and proper algebraic

manipulation the foregoing constants may be put directly in terms of the dimensionless parameters and the number of subdivisions of the matrix and fluid streams, giving

$$K_1 = 2 \left[1 + \frac{1}{\frac{C_{\min}}{C_{\max}} \frac{C_r}{C_{\min}} \frac{N_x}{N_r}} + \frac{2 N_r (hA)'}{NTU_0 [1 + (hA)'] \frac{C_{\min}}{C_{\max}}} \right]^{-1} \quad [8a]$$

$$K_2 = 2 \left[1 + \frac{C_{\min}}{C_{\max}} \frac{C_r}{C_{\min}} \frac{N_x}{N_r} + \frac{2 N_x (hA)'}{NTU_0 [1 + (hA)'] \frac{C_r}{C_{\min}}} \right]^{-1} \quad [8b]$$

$$K_3 = 2 \left[1 + \frac{1}{\frac{C_r}{C_{\min}} \frac{N_n}{N_r}} + \frac{2 N_r}{NTU_0 [1 + (hA)']} \right]^{-1} \quad [8c]$$

$$K_4 = 2 \left[1 + \frac{C_r}{C_{\min}} \frac{N_n}{N_r} + \frac{2 N_n \frac{C_r}{C_{\min}}}{NTU_0 [1 + (hA)']} \right]^{-1} \quad [8d]$$

Now consider the schematic representation of the composite heat exchanger made up of elements as shown in Fig. 3. For illustration the fluid streams and the matrix stream have each been divided into three equal substreams to form the elements. The double lines represent the area of the seals. It should be noted that the left edge is physically the same as the right edge and therefore the matrix inlet temperature of a particular matrix substream on the left must be identical to the matrix outlet temperature of that substream on the right. This will be referred to later as the reversal condition.

In Fig. 3 where individual element temperatures are designated with the same label it is not implied that they are equal numerically, except

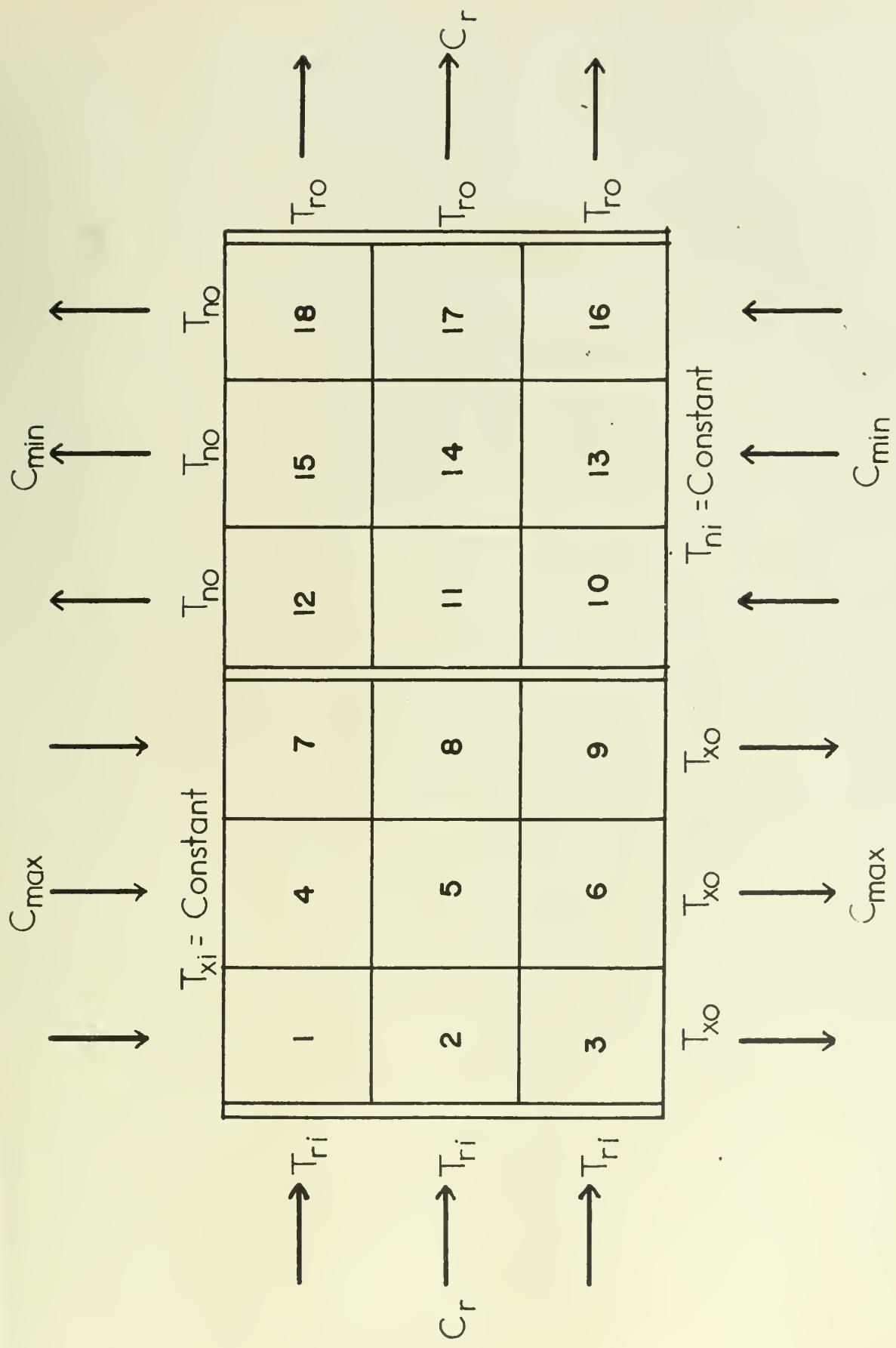


FIG. 3 SCHEMATIC COMPOSITE HEAT EXCHANGER

where indicated as constant across the fluid inlets. This is done to avoid additional subscript notation. Also, it is not to be implied from Fig. 3 that there is necessarily any symmetry with respect to the areas involved.

Now for calculation purposes if the fluid inlet temperatures are given some convenient values, such as one and zero, and a matrix inlet temperature distribution is assumed on the left edge, the remaining temperatures of each element can be calculated in the order indicated by the element numbers by repetitive use of equations [6] and then [7]. If the temperature distribution assumed on the left edge was correct then it would be duplicated on the right, i.e., the reversal condition was fulfilled, and the problem would be solved for the particular set of parameters used. If, however, this is not the case, then the resulting temperature distribution on the right is now used on the left and the procedure repeated until the reversal condition is met.

After several iterations of this type the cyclic character of the calculations for each element and column on a side becomes apparent and the beauty of automatic computing equipment is fully appreciated. It can be seen that if, in the process of computing the outlet temperatures of element one, the T_{x0} is put where the T_{x1} was and a control number increased by one so that the T_{r1} of element two will be used next the same sequence of operations can now be used to compute element two, and so on down the first column of elements. Also, if the T_{r0} of each element is put where its T_{r1} was, then upon reaching the end of the column and "zeroing" the above control number the same cyclic operation can be used for the next column. A second control number is used to indicate when one side is finished and then a similar procedure is followed on

the other side. In this scheme it is only necessary to provide storage for the T_{ri} and T_{ro} of each row, which will be used for comparison purposes to determine if the reversal condition is met, and the T_{xo} of each column on one side and the T_{no} of each column on the other side, which will be used to make a heat balance and compute the effectiveness when the reversal condition is met.

That the reversal condition eventually will be met by this iterative scheme can be shown (Appendix 1) to depend on the physical conditions of the problem and the second law of thermodynamics. This will be called the condition for convergence and for this problem, using equal subdivisions of each stream, it may be stated in the following form:

$$NTU_o [1 + 1/(hA)'] C_{min}/C_{max} \leq 2N \dots \dots \dots [9]$$

Accepting for the moment the idealizations and assumptions made, then the accuracy of the solution obtained, by the very nature of a finite-difference process, depends only on the number of subdivisions used. It is apparent from condition [9] that a greater number of subdivisions will also enhance the convergence.

A considerable number of subdivisions can be handled by automatic computing equipment to obtain a certain accuracy and aid convergence, but the number of subdivisions used must be arrived at by a compromise between the accuracy desired and time available. The easiest way to illustrate the accuracy obtained for a given degree of subdivision is to solve the problem for several values of subdivision and extrapolate the results obtained to zero element area. As an example, consider the following results for the indicated parameter values:

$$C_{min}/C_{max} = 0.90, \quad NTU_o = 6, \quad C_r/C_{min} = (hA)' = 1.0$$

<u>No. subdivisions</u>	<u>Effectiveness</u>
8	0.7816
16	0.7800
32	0.7796

These results are plotted versus element area and extrapolated to zero in Fig. 3a where it is found that the indicated effectiveness is 0.7795. This correction is not constant for all ranges of parameters, being largest for low values of C_r/C_{min} and high values of NTU_o , the foregoing example being typical. The procedure used in this work was to calculate Tables 1 through 9 with 16 subdivisions, then recalculate Tables 1 and 9 with 32 subdivisions. Table 9 was calculated to show the effect of varying the parameter $(hA)'$, and it also contains representative values from the other tables so that with these data all of the tabulated data were extrapolated to zero element area.

The computer used has the equivalent of nine significant decimal figure accuracy; it was specified that the reversal condition be fulfilled to only five figures. Of more significance than this was the final requirement that before a solution was considered as being obtained the heat balance error, which is a direct measure of error in the effectiveness introduced by accuracy of computation, be less than 0.024 per cent.

The remainder of the computer program was essentially a system of "feeding" it the next set of parameters after each solution. Appendix 2 describes the computer program and its various modifications in some detail. Several interesting features incorporated in it are:

- a. Up to 64 subdivisions could be accommodated without external memory storage simply by modifying certain control commands.

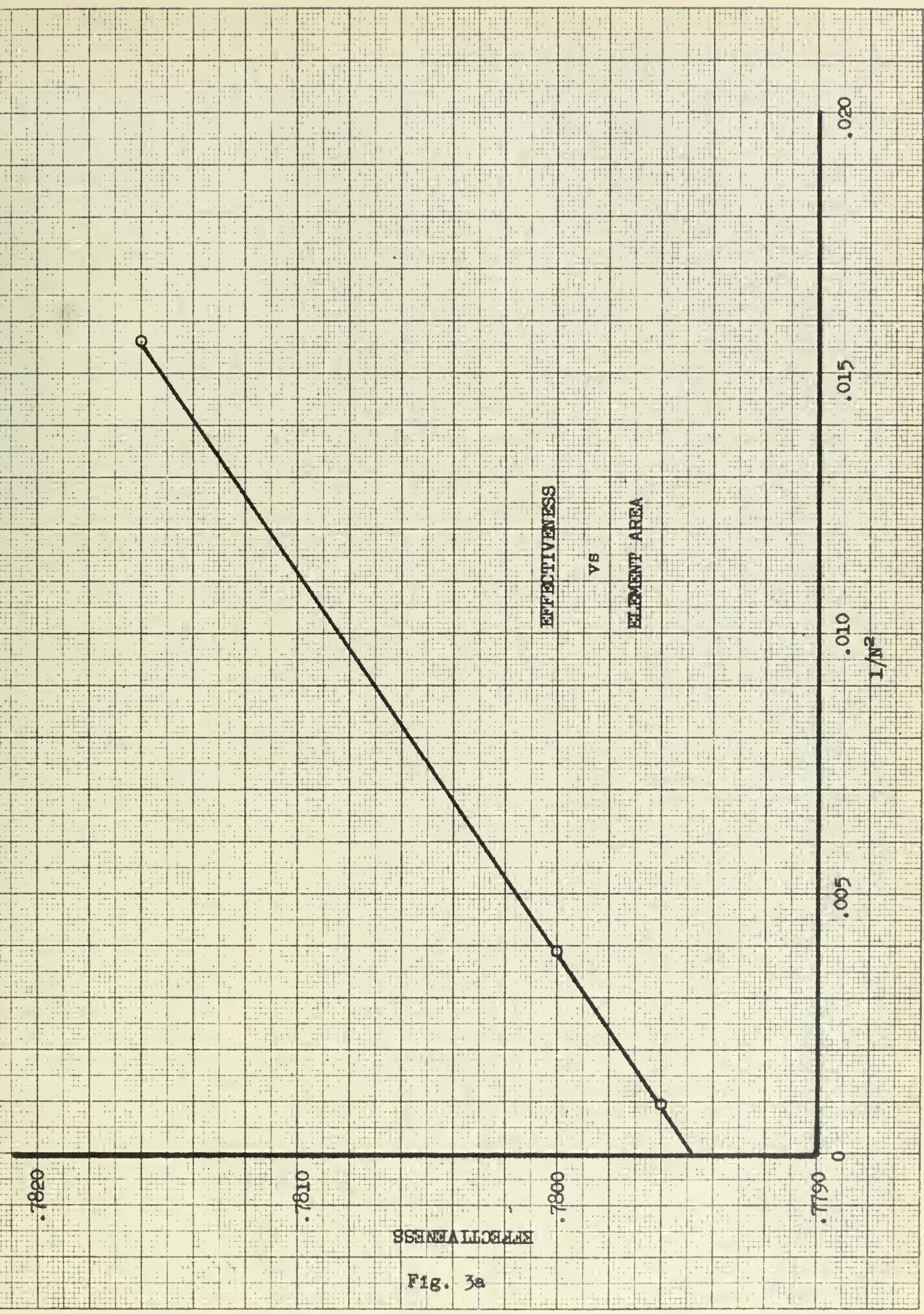


Fig. 3a

- b. Final matrix temperature distributions were punched on cards for future use.
- c. A logical choice of initial estimate for matrix temperature distribution for each new problem was made from data available from work up to that point.
- d. Optional read out of all or selected temperatures was provided.

3. Comparison of Results

The results obtained are reported in tabular (Tables 1 - 9) and graphical form (Figs. 4 - 11). Several general observations can be made from the data:

a. As C_{\min}/C_{\max} decreases, for given values of the other parameters, the effectiveness increases. This is predicted by the limiting solution for $C_r/C_{\min} = \infty$, equation [1], and was pointed out in the illustrative problem of reference (2). This is illustrated in Fig. 12.

b. By using NTU_o as defined in the nomenclature, which contains $(hA)'$ explicitly, the influence of $(hA)'$ at a given NTU_o is very small for $C_{\min}/C_{\max} \geq 0.9$. This conclusion was also reached by Coppage and London (2) in their evaluation of the work by Hausen (6), Saunders and Smoleniec (7), J.E. Johnson (8), and Iliffe (10). For $C_{\min}/C_{\max} < 0.9$, however, the influence of $(hA)'$ becomes increasingly pronounced, and the variation for $(hA)' = \text{unity}$ and 0.25 may be as much as seven per cent for $C_{\min}/C_{\max} = 0.1$. See Table 9.

c. The difference in effectiveness for $C_r/C_{\min} \geq 5$ and the value given by the limiting solution for $C_r/C_{\min} = \infty$ is very small. This may be seen from inspection of Tables 1 through 8 or Figs. 4 through 11.

R.W. Johnson (11) used much the same method employed here but had limited access to a computer and was therefore able to calculate only a relatively small number of the points reported here. Also due to limited computer time, at most ten and in some cases only five subdivisions were used with no extrapolation to zero element area correction. Allowing for this the corresponding results are practically identical.

Coppage and London (2) recommended the results of J.E. Johnson (9) which were obtained for $C_{\min}/C_{\max} = \text{unity}$. These results, in terms of

the parameters used here, were later incorporated in reference (12) with extrapolation to $C_{\min}/C_{\max} = 0.7$. The variation in corresponding values of Table 1 of this work and Table 6, page II-32, of reference (12) which are for $C_{\min}/C_{\max} = \text{unity}$ is less than one per cent. Only when $C_{\min}/C_{\max} = 0.7$ and $C_r/C_{\min} = \text{unity}$, where the extrapolation method used was admitted to be most uncertain, is the variation slightly more than two per cent.

An expression for effectiveness in closed form may be desirable for certain uses such as computer analysis of plant performance. A relatively simple empirical formulation is suggested in reference (12) for $C_{\min}/C_{\max} \geq 0.7$ and $C_r/C_{\min} \geq 2$ and is of the form

$$\frac{E_{\infty} - E}{E_{\infty}} = \frac{\Delta E}{E_{\infty}} = K_5 (C_r/C_{\min})^m \dots \dots \dots [10]$$

where $K_5 = 1/9$, $m = -2$, and E_{∞} is determined from equation [1]. Such a form cannot be applicable for all values of parameters with single values of K_5 and m , but it agrees reasonably well with the results reported here for the ranges specified. It has been found that with slight variation of K_5 and m very close agreement can be obtained in a small range of parameters which may be of particular interest. As an example, in the range

$$1.0 \geq C_{\min}/C_{\max} \geq 0.90$$

$$1.25 \leq C_r/C_{\min} \leq 5.0$$

$$3.0 \leq NTU_0 \leq 9.0$$

with $K_5 = 1/9$ and $m = -1.87$, the error is less than one per cent. Fig. 13 illustrates the method used to determine K_5 and m in this range.

4. Limitations

The most questionable of the idealizations made is that of zero matrix conductivity, particularly in the direction of fluid flow. Although the effect of matrix conductivity could be included in the finite-difference equations the additional degrees of freedom would make adequate coverage of all the parameters extremely voluminous and time consuming. If for a particular exchanger design the matrix conductivity was considered important then the problem could be solved individually on a computer with adequate capacity. A simpler alternative is to neglect the conductivity and correct the effectiveness for this effect. From theoretical considerations Professor A.L. London has shown (13) that a correction factor for conductivity in the direction of fluid flow for approximately equal fluid capacity rates, of the form

should result in a somewhat pessimistic prediction of the reduction in effectiveness. (A in this case is the matrix cross-sectional area available for heat conduction in the direction of fluid flow on both sides, and L is the flow length). Schultz (14) has solved for the effectiveness as a function of the conduction parameter in equation [11] for three cases. In the nomenclature used here, these three cases are for the following conditions:

- a. $C_{\min} = C_{\max}$, C_r/C_{\min} very large, k finite
- b. $C_{\min} = C_{\max}$, $C_r/C_{\min} = \infty$, k finite, regenerator subdivided into sections in direction of fluid flow
- c. $C_{\min} = C_{\max}$, C_r/C_{\min} finite, $k = \infty$, regenerator subdivided into sections in direction of fluid flow.

From the curves in reference (14) which allow direct comparison with the results presented here it is found that for

$$\frac{kA}{LC} \leq 0.02 \dots \dots \dots \dots \dots [12]$$

the reduction in effectiveness is at most about one per cent. Additional work, not available at the time of writing, concerning the influence of conduction is reported in reference (15).

The effect of leakage can be calculated separately resulting in a correction to the effectiveness without leakage. This has been shown to be small for as much ten per cent leakage (1).

5. Summary and Conclusions

The numerical procedure used in this work assumes no symmetry, either with respect to heat capacity rates of the fluids or area ratios of the two sides, and therefore is not limited in this respect. The ranges of parameters which have been covered effectively are

$$1.0 \geq C_{\min}/C_{\max} \geq 0.10$$

$$1.0 \leq C_r/C_{\min} \leq \infty$$

$$1.0 \leq NTU_0 \leq 10$$

$$1.0 \geq (hA)' \geq 0.25$$

The results are tabulated at frequent enough intervals and over sufficient ranges of the parameters to include the gas-turbine regenerator application as well as other applications, such as intercooling, where the values of the parameters might be quite different.

The error introduced by the finite-difference type of solution has been minimized by use of sufficient subdivision and further extrapolation to zero element area so that four significant figures in the results are justified.

The effects of deviations from the assumed idealizations have not been included in the results.

Even though the results obtained cannot be made to fit a completely general closed form, a relatively simple expression for effectiveness can be obtained which will give good agreement over a reasonable range of parameters.

TABLE 1 PERIODIC-FLOW HEAT EXCHANGER EFFECTIVENESS

$$C_{\min}/C_{\max} = 1.0$$

$$(hA)' = 1.0$$

NTU _o	C _r /C _{min}						
	1	1.25	1.5	2	3	5	∞
0.5	0.3221		0.3282	0.3304		0.3329	0.3333
1.0	0.4665	0.4779	0.4845	0.4912	0.4960	0.4985	0.5000
1.5	0.5478		0.5757	0.5861		0.5978	0.6000
2.0	0.6007	0.6231	0.6360	0.6491	0.6587	0.6638	0.6667
2.5	0.6385		0.6792	0.6941		0.7110	0.7142
3.0	0.6672	0.6959	0.7119	0.7280	0.7400	0.7463	0.7500
3.5	0.6900		0.7377	0.7546		0.7739	0.7778
4.0	0.7086	0.7410	0.7586	0.7760	0.7889	0.7959	0.8000
4.5	0.7242		0.7760	0.7937		0.8139	0.8181
5.0	0.7375	0.7723	0.7907	0.8086	0.8217	0.8290	0.8333
5.5	0.7491		0.8034	0.8213		0.8417	0.8461
6.0	0.7592	0.7956	0.8144	0.8323	0.8453	0.8526	0.8571
6.5	0.7682		0.8241	0.8419		0.8621	0.8667
7.0	0.7763	0.8138	0.8327	0.8504	0.8632	0.8704	0.8750
7.5	0.7835		0.8404	0.8579		0.8777	0.8823
8.0	0.7901	0.8285	0.8473	0.8647	0.8772	0.8842	0.8889
8.5	0.7961		0.8536	0.8708		0.8900	0.8947
9.0	0.8017	0.8407	0.8593	0.8763	0.8884	0.8953	0.9000
9.5	0.8068		0.8646	0.8814		0.9002	0.9047
10.0	0.8115	0.8510	0.8694	0.8860	0.8978	0.9045	0.9091

TABLE 2 PERIODIC-FLOW HEAT EXCHANGER EFFECTIVENESS

 $C_{min}/C_{max} = 0.95$ $(hA)' = 1.0$

NTU _o	C _r /C _{min}						
	1	1.25	1.5	2	3	5	∞
1	0.4714	0.4833	0.4901	0.4970	0.5021	0.5047	0.5063
2	0.6082	0.6319	0.6454	0.6592	0.6693	0.6747	0.6778
3	0.6760	0.7064	0.7234	0.7406	0.7533	0.7600	0.7640
4	0.7183	0.7528	0.7715	0.7903	0.8039	0.8113	0.8158
5	0.7477	0.7849	0.8047	0.8240	0.8379	0.8455	0.8503
6	0.7698	0.8090	0.8292	0.8484	0.8624	0.8700	0.8750
7	0.7872	0.8277	0.8481	0.8672	0.8808	0.8884	0.8934
8	0.8012	0.8429	0.8633	0.8820	0.8953	0.9027	0.9077
9	0.8129	0.8554	0.8757	0.8940	0.9069	0.9142	0.9191
10	0.8229	0.8661	0.8861	0.9040	0.9165	0.9235	0.9284

TABLE 3 PERIODIC-FLOW HEAT EXCHANGER EFFECTIVENESS

 $C_{\min}/C_{\max} = 0.90$ $(hA)^{\prime} = 1.0$

NTU _o	C_r/C_{\min}						
	1	1.25	1.5	2	3	5	∞
1	0.4763	0.4887	0.4958	0.5030	0.5082	0.5110	0.5126
2	0.6156	0.6405	0.6547	0.6693	0.6800	0.6856	0.6889
3	0.6846	0.7167	0.7347	0.7530	0.7664	0.7735	0.7777
4	0.7275	0.7642	0.7841	0.8040	0.8185	0.8263	0.8310
5	0.7573	0.7970	0.8181	0.8385	0.8534	0.8615	0.8664
6	0.7795	0.8216	0.8432	0.8636	0.8784	0.8865	0.8916
7	0.7970	0.8407	0.8625	0.8827	0.8972	0.9051	0.9102
8	0.8112	0.8561	0.8780	0.8978	0.9117	0.9195	0.9246
9	0.8230	0.8689	0.8906	0.9099	0.9234	0.9309	0.9359
10	0.8331	0.8797	0.9012	0.9199	0.9329	0.9401	0.9450

TABLE 4 PERIODIC-FLOW HEAT EXCHANGER EFFECTIVENESS

$$C_{\min}/C_{\max} = 0.80$$

$$(hA)' = 1.0$$

NTU _o	C _r /C _{min}					∞
	1	1.25	1.5	2	3	
1	0.4861	0.4994	0.5071	0.5149	0.5207	0.5254
2	0.6299	0.6573	0.6730	0.6891	0.7011	0.7109
3	0.7007	0.7364	0.7565	0.7769	0.7919	0.8043
4	0.7443	0.7853	0.8077	0.8299	0.8461	0.8597
5	0.7743	0.8190	0.8427	0.8655	0.8818	0.8957
6	0.7967	0.8440	0.8683	0.8910	0.9069	0.9206
7	0.8140	0.8634	0.8879	0.9101	0.9253	0.9386
8	0.8279	0.8788	0.9033	0.9249	0.9392	0.9518
9	0.8394	0.8914	0.9157	0.9365	0.9500	0.9619
10	0.8492	0.9021	0.9260	0.9460	0.9586	0.9696

TABLE 5 PERIODIC-FLOW HEAT EXCHANGER EFFECTIVENESS

$$C_{\min}/C_{\max} = 0.70$$

$$(hA)^{\prime} = 1.0$$

NTU _o	C _r /C _{min}					1 ∞
	1	1.25	1.5	2	3	
1	0.4959	0.5104	0.5187	0.5271	0.5333	0.5384
2	0.6437	0.6737	0.6910	0.7087	0.7219	0.7326
3	0.7155	0.7548	0.7770	0.7995	0.8160	0.8295
4	0.7590	0.8044	0.8291	0.8535	0.8710	0.8855
5	0.7887	0.8382	0.8643	0.8891	0.9063	0.9207
6	0.8104	0.8629	0.8895	0.9139	0.9303	0.9439
7	0.8272	0.8818	0.9085	0.9320	0.9473	0.9598
8	0.8405	0.8967	0.9232	0.9456	0.9596	0.9709
9	0.8514	0.9087	0.9348	0.9560	0.9687	0.9788
10	0.8606	0.9188	0.9443	0.9642	0.9756	0.9845

TABLE 6 PERIODIC-FLOW HEAT EXCHANGER EFFECTIVENESS

 $C_{\min}/C_{\max} = 0.50$ $(hA)^{\prime} = 1.0$

NTU _o	C _r /C _{min}					∞
	1	1.25	1.5	2	3	
1	0.5156	0.5322	0.5417	0.5515	0.5588	0.5647
2	0.6690	0.7045	0.7249	0.7461	0.7618	0.7746
3	0.7402	0.7869	0.8133	0.8399	0.8590	0.8744
4	0.7817	0.8353	0.8645	0.8928	0.9122	0.9274
5	0.8089	0.8670	0.8975	0.9256	0.9436	0.9572
6	0.8284	0.8894	0.9202	0.9469	0.9629	0.9745
7	0.8431	0.9062	0.9366	0.9615	0.9752	0.9847
8	0.8546	0.9191	0.9489	0.9717	0.9833	0.9908
9	0.8639	0.9293	0.9582	0.9789	0.9886	0.9944
10	0.8716	0.9376	0.9655	0.9842	0.9921	0.9966

TABLE 7 PERIODIC-FLOW HEAT EXCHANGER EFFECTIVENESS

$$C_{\min}/C_{\max} = 0.30$$

$$(hA)' = 1.0$$

NTU _o	C _r /C _{min}					∞
	1	1.25	1.5	2	3	
1	0.5346	0.5538	0.5648	0.5761	0.5846	0.5915
2	0.6902	0.7315	0.7556	0.7804	0.7988	0.8136
3	0.7577	0.8114	0.8422	0.8729	0.8945	0.9110
4	0.7951	0.8558	0.8895	0.9214	0.9420	0.9566
5	0.8191	0.8838	0.9183	0.9490	0.9670	0.9787
6	0.8360	0.9031	0.9374	0.9658	0.9808	0.9895
7	0.8487	0.9171	0.9506	0.9764	0.9885	0.9948
8	0.8588	0.9278	0.9602	0.9835	0.9930	0.9974
9	0.8671	0.9363	0.9673	0.9881	0.9955	0.9987
10	0.8740	0.9433	0.9729	0.9913	0.9971	0.9994

TABLE 8 PERIODIC-FLOW HEAT EXCHANGER EFFECTIVENESS

$$C_{\min}/C_{\max} = 0.10$$

$$(hA)' = 1.0$$

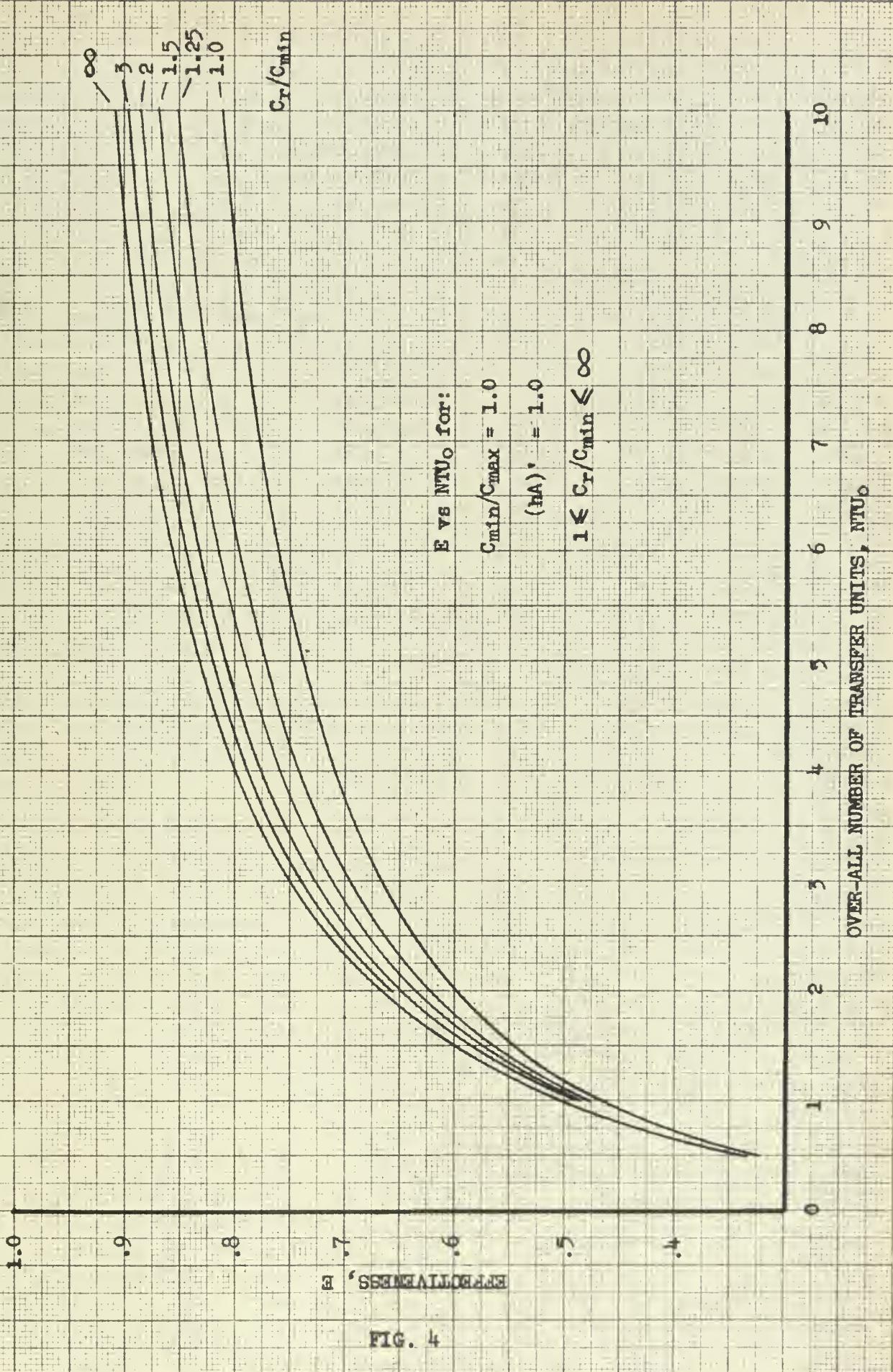
NTU _o	C _r /C _{min}					
	1	1.25	1.5	2	3	∞
1	0.5534	0.5752	0.5877	0.6008	0.6104	0.6186
2	0.7070	0.7542	0.7820	0.8109	0.8319	0.8487
3	0.7680	0.8279	0.8630	0.8981	0.9219	0.9391
4	0.8009	0.8667	0.9042	0.9397	0.9615	0.9753
5	0.8223	0.8908	0.9283	0.9618	0.9802	0.9900
6	0.8378	0.9075	0.9440	0.9745	0.9892	0.9959
7	0.8498	0.9200	0.9550	0.9824	0.9939	0.9983
8	0.8595	0.9298	0.9632	0.9874	0.9964	0.9993
9	0.8675	0.9376	0.9694	0.9908	0.9977	0.9997
10	0.8743	0.9441	0.9743	0.9932	0.9984	0.9999

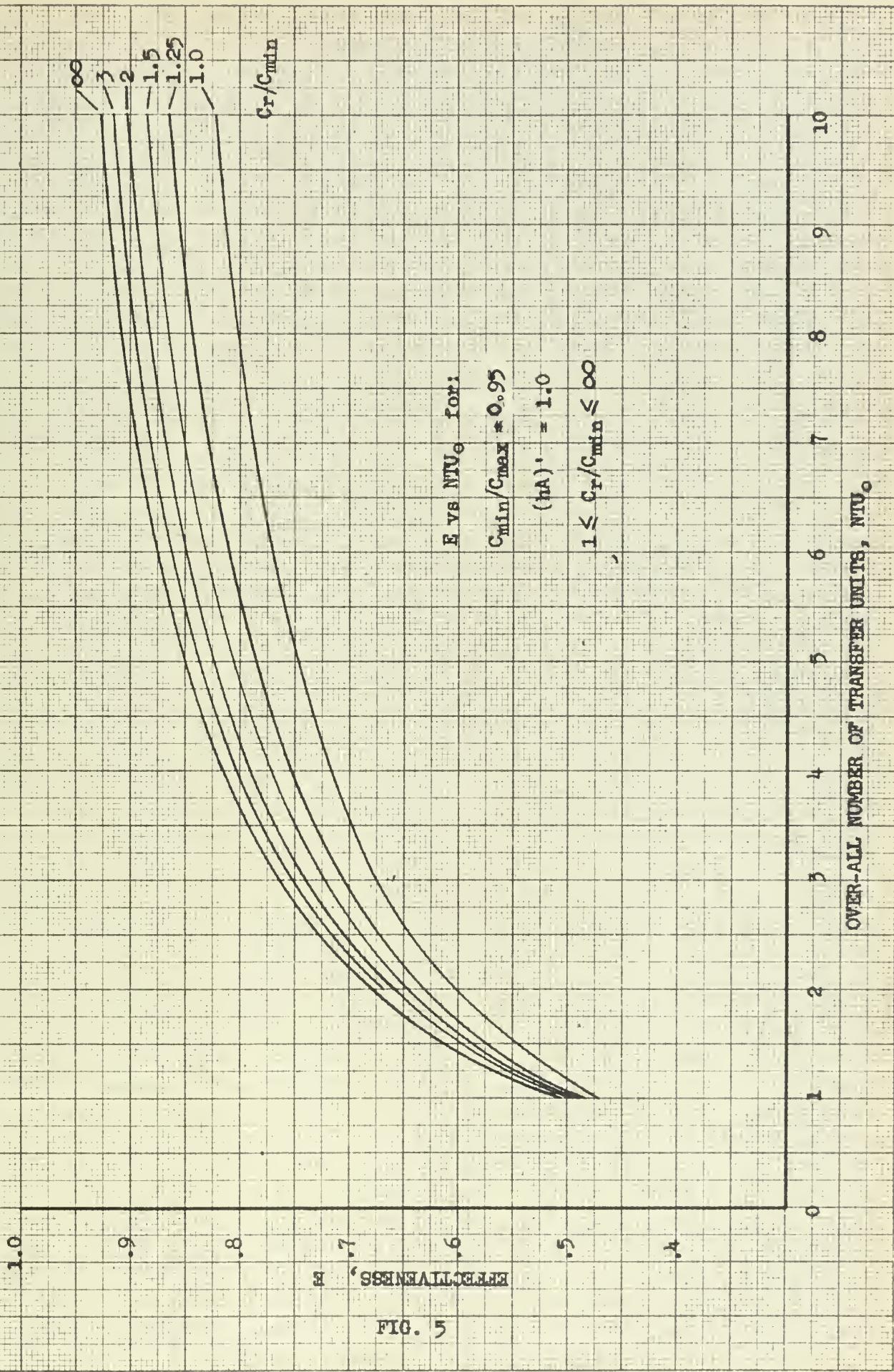
TABLE 9 PERIODIC-FLOW HEAT EXCHANGER EFFECTIVENESS

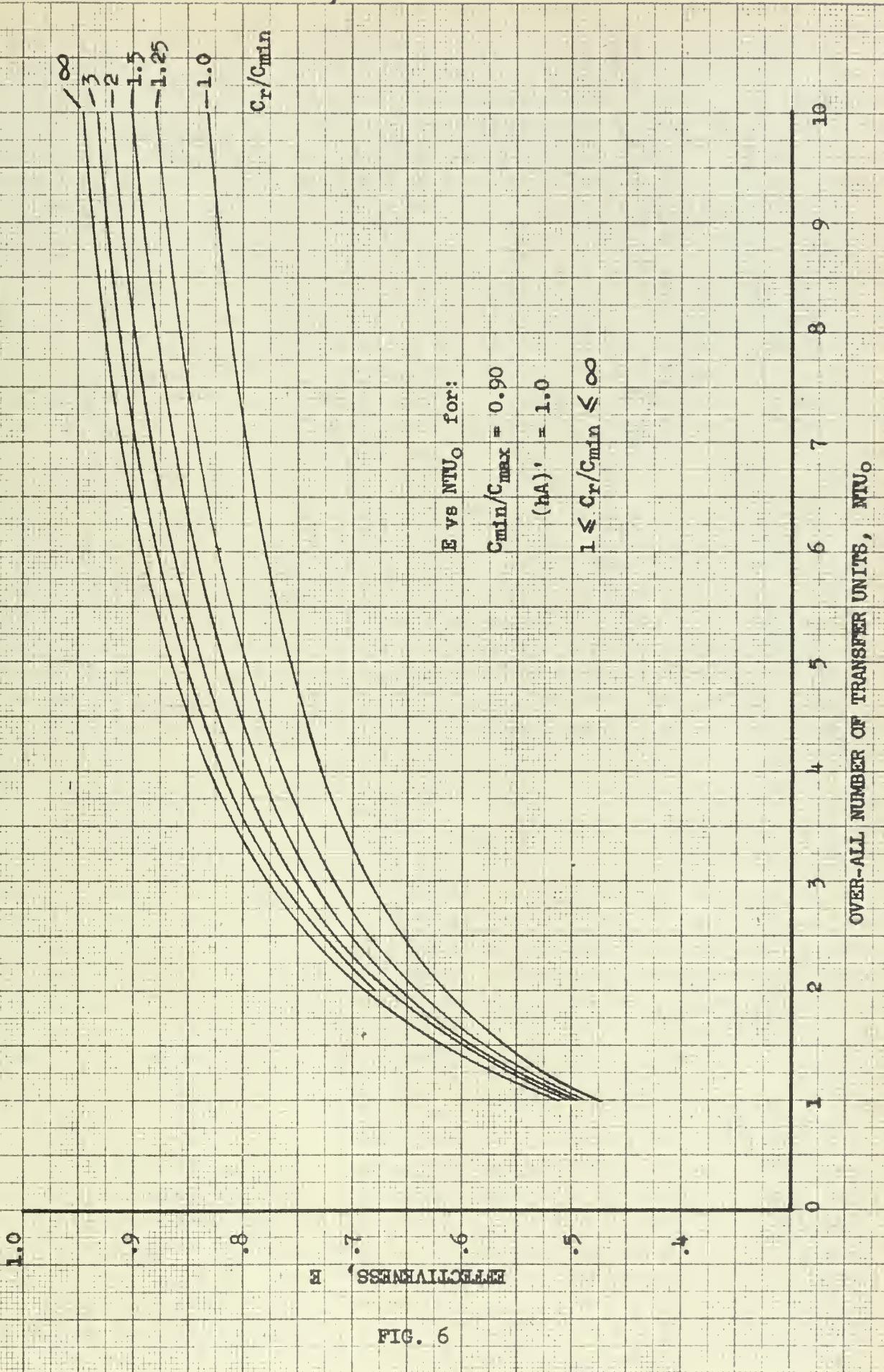
NTU _o	(hA) ¹	C_r/C_{min}					
		1	2	1	2	1	2
		$C_{min}/C_{max} = 1.0$		$C_{min}/C_{max} = 0.95$		$C_{min}/C_{max} = 0.90$	
3	1.00	0.6672	0.7280	0.6760	0.7406	0.6846	0.7530
3	0.50	0.6676	0.7282	0.6752	0.7404	0.6826	0.7524
3	0.25	0.6684	0.7284	0.6751	0.7404	0.6813	0.7521
6	1.00	0.7592	0.8323	0.7698	0.8484	0.7795	0.8636
6	0.50	0.7597	0.8323	0.7687	0.8481	0.7769	0.8628
6	0.25	0.7608	0.8324	0.7685	0.8478	0.7751	0.8622
9	1.00	0.8017	0.8763	0.8129	0.8940	0.8230	0.9099
9	0.50	0.8021	0.8763	0.8115	0.8936	0.8198	0.9091
9	0.25	0.8032	0.8763	0.8111	0.8933	0.8176	0.9084
		$C_{min}/C_{max} = 0.80$		$C_{min}/C_{max} = 0.70$		$C_{min}/C_{max} = 0.50$	
3	1.00	0.7007	0.7769	0.7155	0.7995	0.7402	0.8399
3	0.50	0.6960	0.7752	0.7075	0.7964	0.7245	0.8331
3	0.25	0.6919	0.7742	0.7001	0.7943	0.7099	0.8270
6	1.00	0.7967	0.8910	0.8104	0.9139	0.8284	0.9469
6	0.50	0.7902	0.8888	0.8000	0.9103	0.8102	0.9398
6	0.25	0.7848	0.8872	0.7908	0.9074	0.7953	0.9334
9	1.00	0.8394	0.9365	0.8514	0.9560	0.8639	0.9789
9	0.50	0.8323	0.9344	0.8402	0.9529	0.8463	0.9735
9	0.25	0.8263	0.9328	0.8306	0.9503	0.8327	0.9684

TABLE 9 CONTINUED

NTU ₀	(hA) ¹	C _r /C _{min}		C _{min} /C _{max} = 0.10	
		1	2	1	2
3	1.00	0.7577	0.8729	0.7680	0.8981
3	0.50	0.7338	0.8608	0.7373	0.8787
3	0.25	0.7131	0.8484	0.7136	0.8579
6	1.00	0.8360	0.9658	0.8379	0.9745
6	0.50	0.8130	0.9546	0.8133	0.9597
6	0.25	0.7957	0.9439	0.7957	0.9457
9	1.00	0.8671	0.9881	0.8675	0.9908
9	0.50	0.8472	0.9808	0.8472	0.9821
9	0.25	0.8327	0.9734	0.8326	0.9738







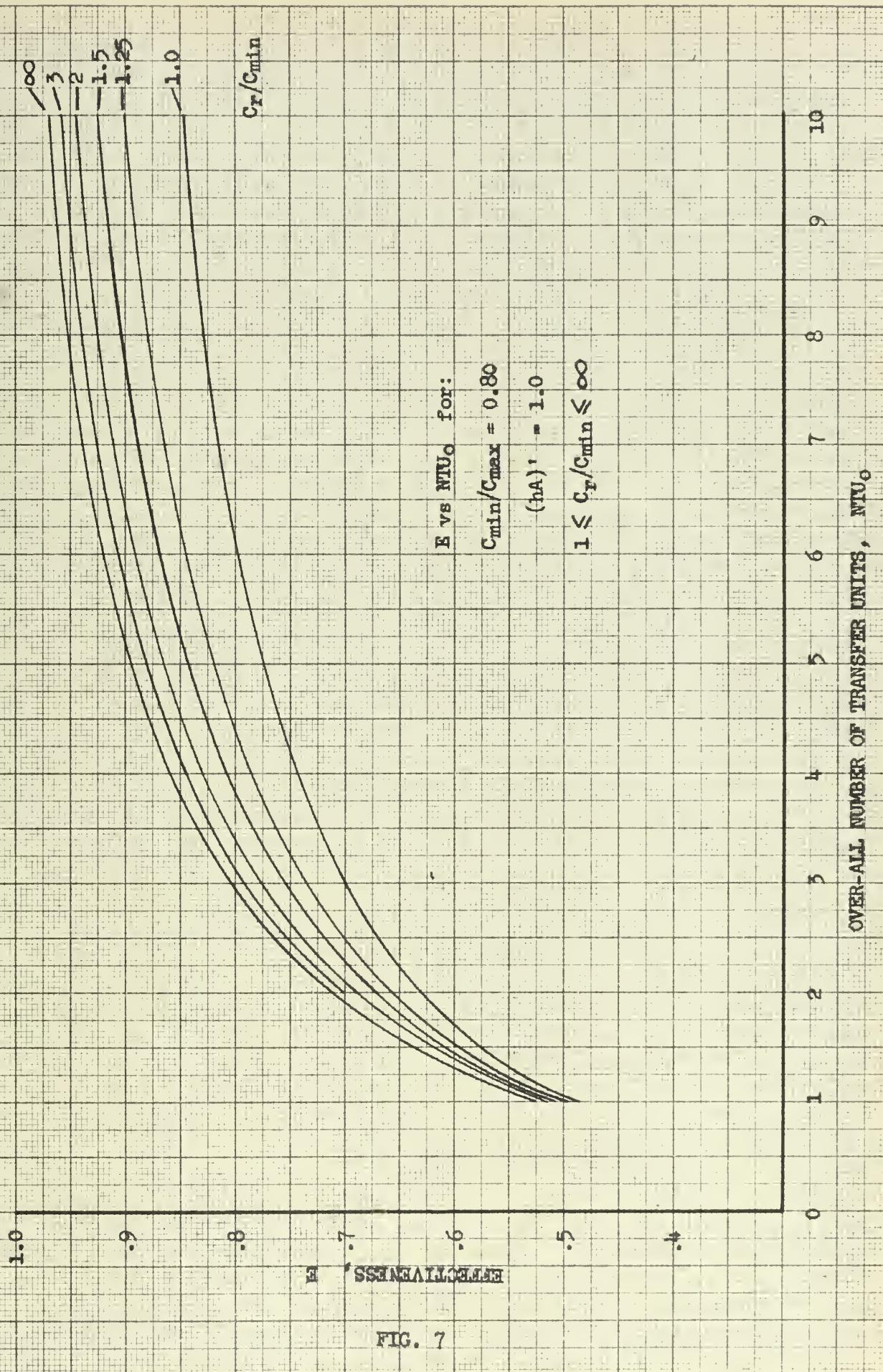


FIG. 7

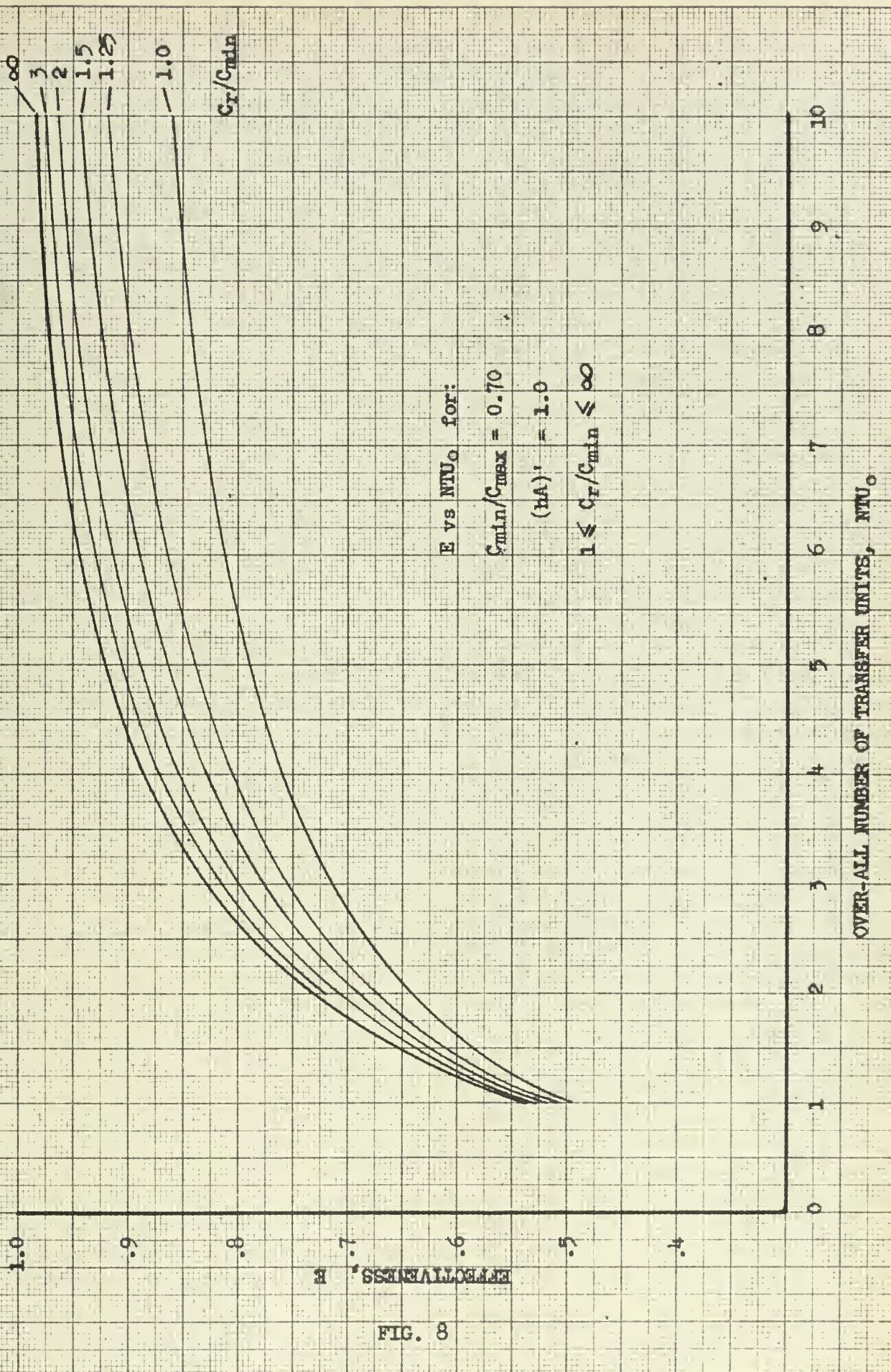


FIG. 8

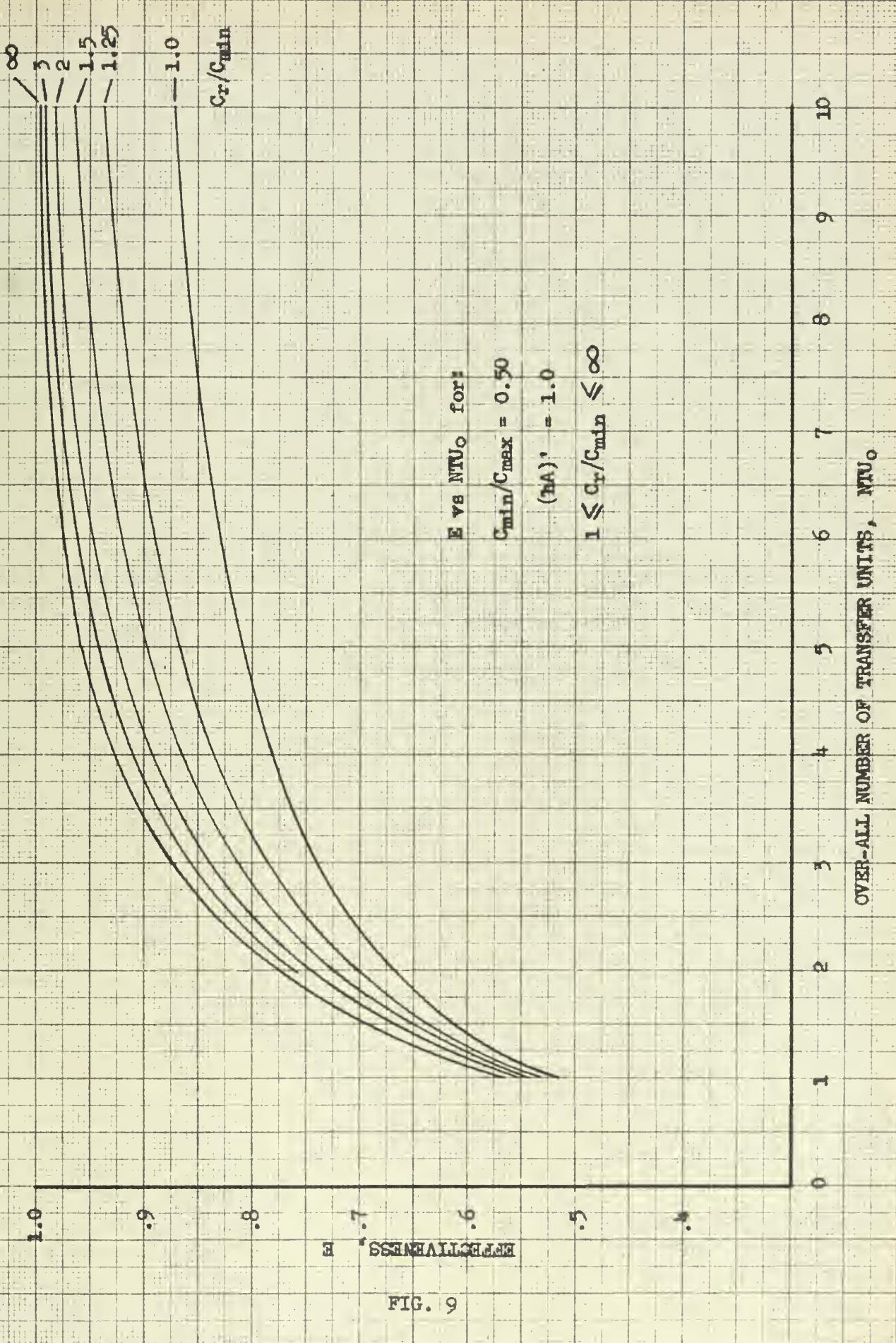


FIG. 9

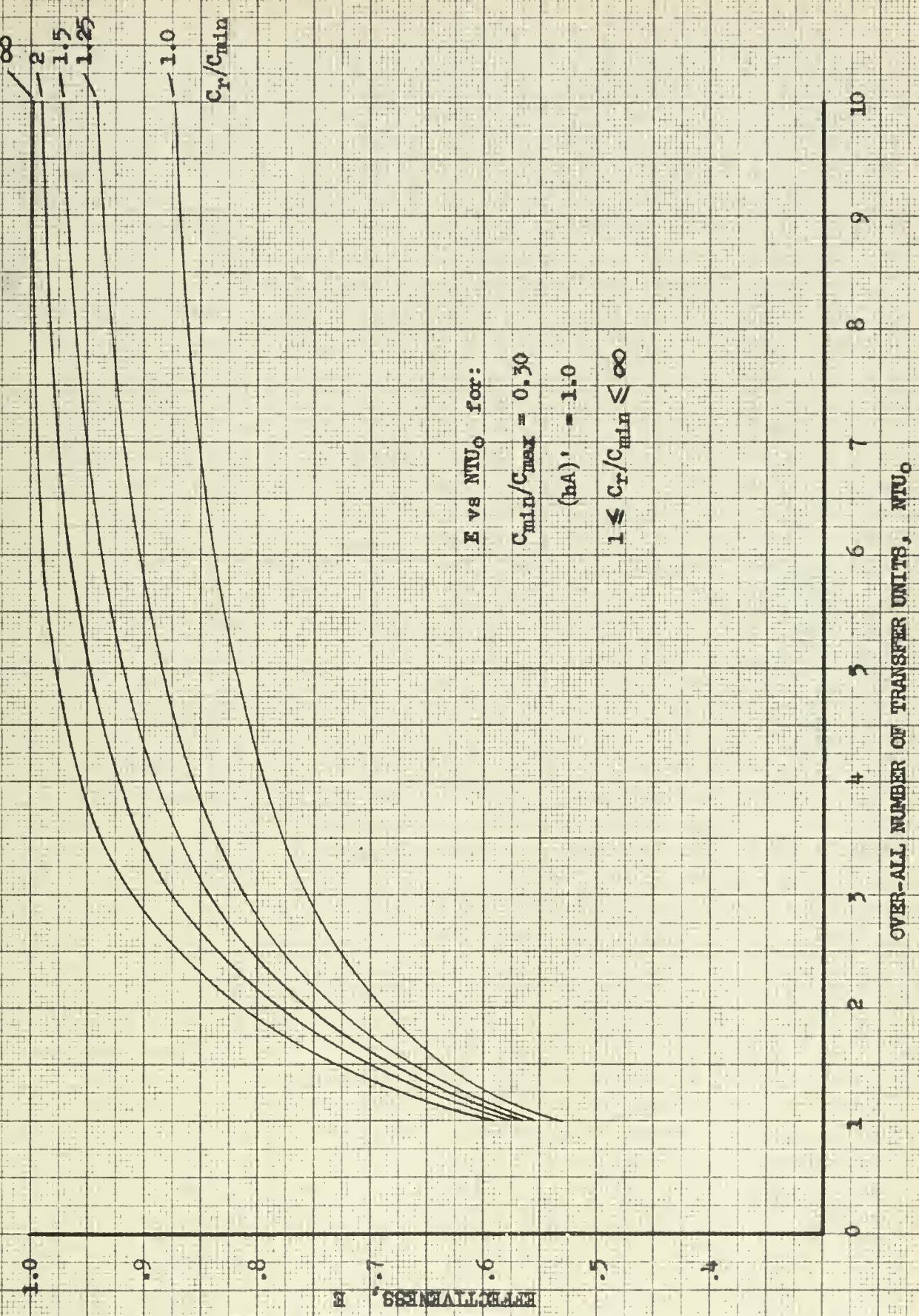


FIG. 10

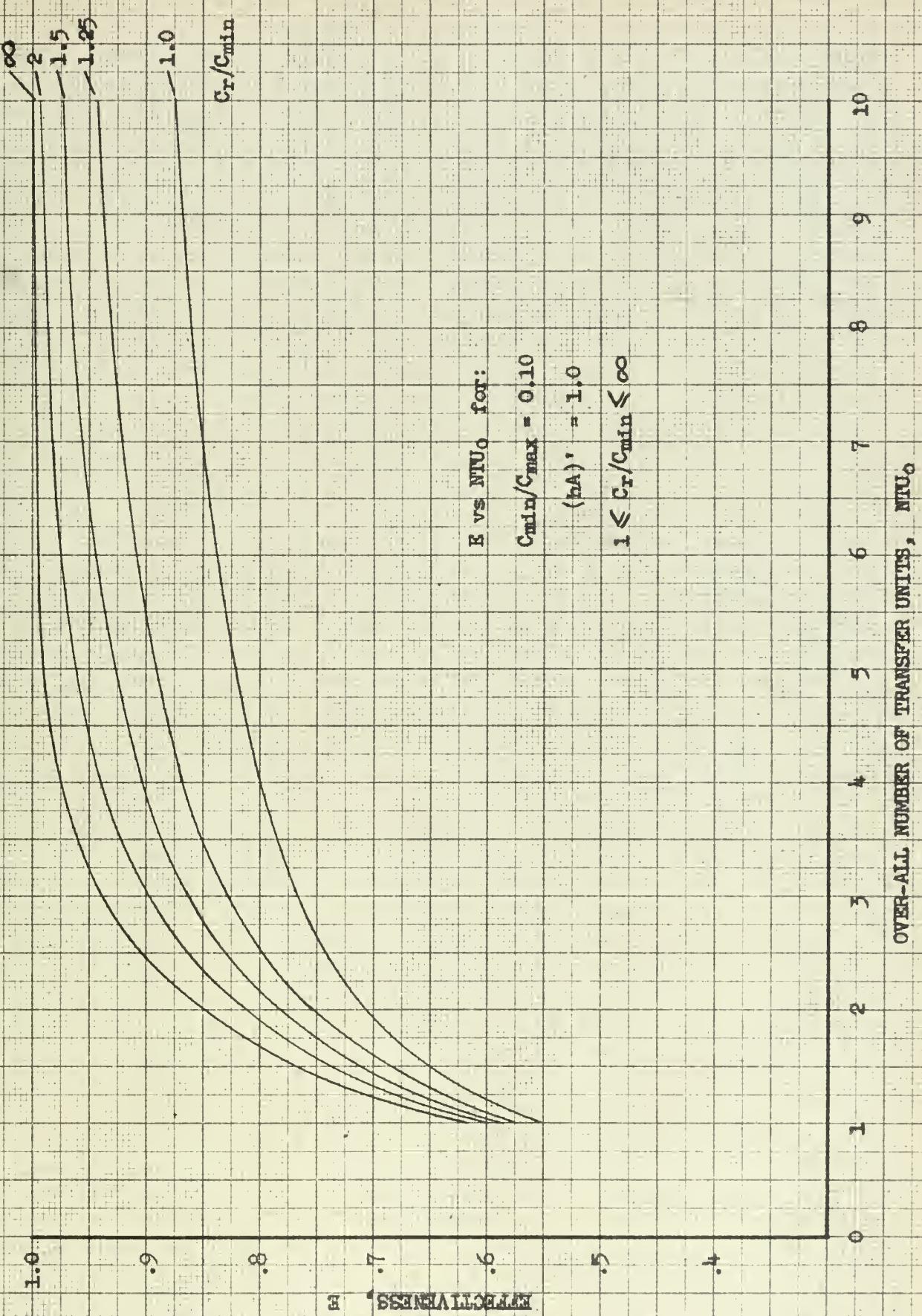
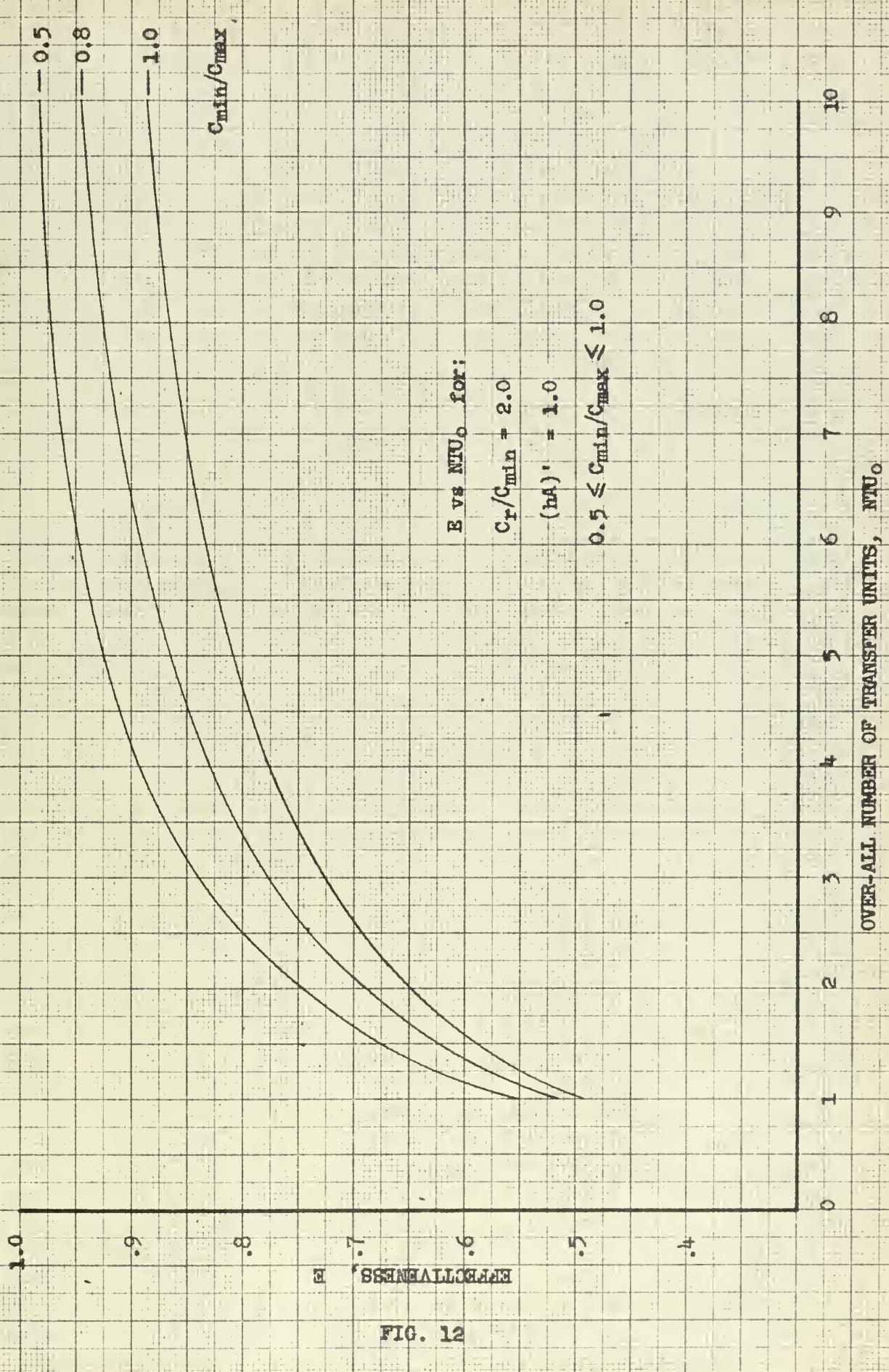


FIG. 11



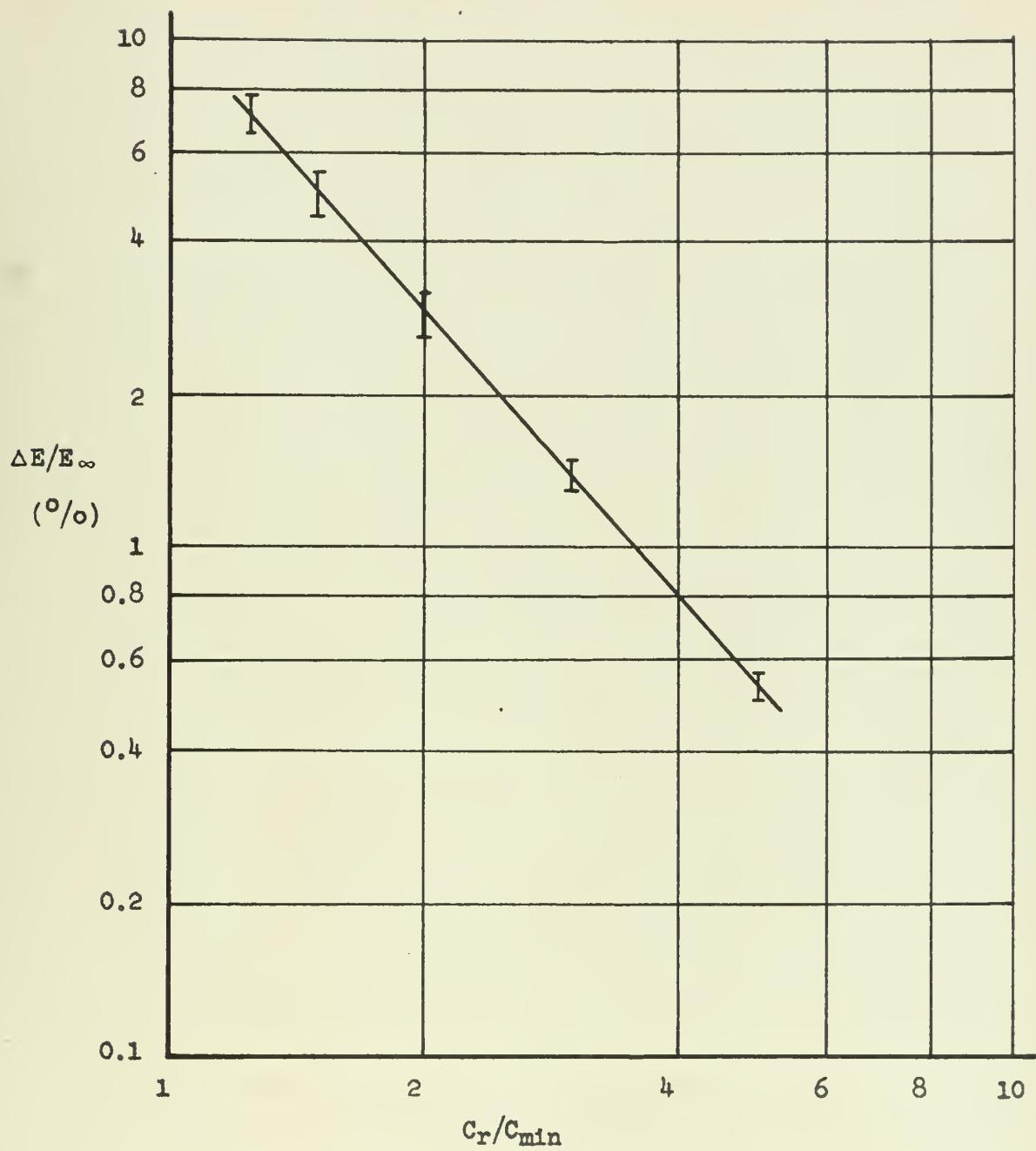


FIG. 13 $\Delta E/E_\infty$ vs C_r/C_{\min} in the range:

$1.0 \geq C_{\min}/C_{\max} \geq 0.90$

$3.0 \leq NTU_0 \leq 9.0$

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9. Appendix 1 - Derivation of Iterative Equations

Equations [2], [3], [4] and [5] concerning the element on the side of C_{\max} , Fig. 2(a), from the body of the thesis are repeated here for convenience.

$$Q = C_{\max} (T_{xi} - T_{xo}) (1/N_x) \dots \dots \dots [2]$$

$$Q = C_r (T_{ro} - T_{ri}) (1/N_r) \dots \dots \dots [3]$$

$$Q = (hA)_x \Delta T_{avg} (1/N_r 1/N_x) \dots \dots \dots [4]$$

$$\Delta T_{avg} = (\gamma_2) (T_{xi} + T_{xo}) - (\gamma_2) (T_{ri} + T_{ro}) \dots \dots \dots [5]$$

If ΔT_{avg} from equation [5] is substituted in equation [4] and the value for Q thus obtained is substituted in equation [2] the following will be obtained:

$$(\gamma_2) (hA)_x (T_{xi} + T_{xo} - T_{ri} - T_{ro}) (1/N_r 1/N_x) = C_{\max} (T_{xi} - T_{xo}) (1/N_x)$$

This may be solved for T_{xo} , giving

$$T_{xo} = \left[\frac{(hA)_x}{2N_r C_{\max} + (hA)_x} \right] (T_{ri} + T_{ro}) + \left[\frac{2N_r C_{\max} - (hA)_x}{2N_r C_{\max} + (hA)_x} \right] T_{xi} \dots [13]$$

In a similar manner T_{ro} may be solved for by using equation [3], giving

$$T_{ro} = \left[\frac{(hA)_x}{2N_x C_r + (hA)_x} \right] (T_{xi} + T_{xo}) + \left[\frac{2N_x C_r - (hA)_x}{2N_x C_r + (hA)_x} \right] T_{ri} \dots [14]$$

At this point four relations can be derived which will be useful in subsequent manipulations to get expressions in terms of the dimensionless parameters. From the definition of NTU_0 it is obvious that:

$$\frac{C_{\min}}{(hA)_n} = \frac{1}{NTU_n} = \frac{1}{NTU_0[1+(hA)'] \quad \dots \quad [15]}$$

An expression for reciprocal NTU_x may be derived by multiplying and dividing it by NTU_n as expressed in equation [15], thus,

$$\begin{aligned} \frac{C_{\max}}{(hA)_x} &= \frac{1}{NTU_x} = \frac{C_{\max}}{(hA)_x} \frac{1}{NTU_0[1+(hA)']} \frac{(hA)_n}{C_{\min}} \\ &= \frac{(hA)'}{NTU_0[1+(hA)']} \frac{C_{\min}/C_{\max}}{\dots \quad [16]} \end{aligned}$$

The other two expressions, the derivations of which are obvious, are as follows:

$$\frac{C_r}{(hA)_n} = \frac{C_r/C_{\min}}{NTU_0[1+(hA)']} \quad \dots \quad [17]$$

and,

$$\frac{C_r}{(hA)_x} = \frac{(hA)' C_r/C_{\min}}{NTU_0[1+(hA)']} \quad \dots \quad [18]$$

To obtain T_{x0} in terms of only the inlet temperatures substitute equation [14] in equation [13] and solve for T_{x0} . Starting with equations [13] and [14] slightly rearranged, the algebra involved follows.

$$\left[\frac{2N_r}{NTU_x} + 1 \right] T_{x0} = T_{ri} + T_{ro} + \left[\frac{2N_r}{NTU_x} - 1 \right] T_{xi} \quad \dots \quad [13a]$$

$$T_{ro} = \left[\frac{1}{\frac{2N_x C_r}{(hA)_x} + 1} \right] (T_{xi} + T_{x0}) + \left[1 - \frac{2}{\frac{2N_x C_r}{(hA)_x} + 1} \right] T_{ri} \quad \dots \quad [14a]$$

Substituting [14a] in [13a],

$$\left[\frac{2N_r}{NTU_x} + 1 - \frac{1}{\frac{2N_x C_r}{(hA)_x} + 1} \right] T_{x0} = \left[\frac{2N_r}{NTU_x} - 1 + \frac{1}{\frac{2N_x C_r}{(hA)_x} + 1} \right] T_{xi} + \left[2 - \frac{2}{\frac{2N_x C_r}{(hA)_x} + 1} \right] T_{ri}$$

Each of the terms in the brackets may be put over the same common denominator which may then be multiplied out, giving

$$\left[\frac{4N_r N_x C_r}{NTU_x (hA)_x} + \frac{2N_r}{NTU_x} + \frac{2N_x C_r}{(hA)_x} + 1 - 1 \right] T_{x0} = \left[\frac{4N_r N_x C_r}{NTU_x (hA)_x} + \frac{2N_r}{NTU_x} - \frac{2N_x C_r}{(hA)_x} - 1 + 1 \right] T_{xi} \\ + \left[\frac{4N_x C_r}{(hA)_x} + 2 - 2 \right] T_{ri}$$

Multiplying through by $(hA)' / 2N_x C_r$ and adding and subtracting $2T_{x1}$ on the right gives

$$\left[\frac{2N_r}{NTU_x} + \frac{C_{max} N_r}{C_r N_x} + 1 \right] T_{x0} = \left[\frac{2N_r}{NTU_x} + \frac{C_{max} N_r}{C_r N_x} + 1 \right] T_{xi} - 2(T_{xi} - T_{ri})$$

Now solving for T_{x0} gives a form convenient for iterative calculations, thus,

$$T_{x0} = T_{xi} - 2 \left[1 + \frac{C_{max} N_r}{C_r N_x} + \frac{2N_r}{NTU_x} \right]^{-1} (T_{xi} - T_{ri}) \dots [19]$$

The term in brackets is constant for a given problem so that

$$T_{x0} = T_{xi} - K_1 (T_{xi} - T_{ri}) \dots \dots \dots [6a]$$

where K_1 may be put directly in terms of the dimensionless parameters giving

$$K_1 = 2 \left[1 + \frac{1}{\frac{C_{min}}{C_{max}} \frac{C_r}{C_{min}} \frac{N_x}{N_r}} + \frac{2N_r (hA)'}{NTU_x [1 + (hA)'] \frac{C_{min}}{C_{max}}} \right]^{-1} \dots \dots [8a]$$

To obtain T_{ro} in terms of only the inlet temperatures substitute equation [13] in equation [14] and solve for T_{ro} . Starting with equations [13] and [14] slightly rearranged, the algebra involved follows.

$$T_{x0} = \left[\frac{1}{\frac{2N_r}{NTU_x} + 1} \right] (T_{ri} + T_{ro}) + \left[1 - \frac{2}{\frac{2N_r}{NTU_x} + 1} \right] T_{xi} \dots \dots [13b]$$

$$\left[\frac{2N_x C_r}{(hA)_x} + 1 \right] T_{ro} = T_{xi} + T_{x_0} + \left[\frac{2N_x C_r}{(hA)_x} - 1 \right] T_{ri} \dots \dots \dots [14b]$$

Substituting [13b] in [14b] ,

$$\left[\frac{2N_x C_r}{(hA)_x} + 1 - \frac{1}{\frac{2N_r}{NTU_x} + 1} \right] T_{ro} = \left[\frac{2N_x C_r}{(hA)_x} - 1 + \frac{1}{\frac{2N_r}{NTU_x} + 1} \right] T_{ri} + \left[2 - \frac{2}{\frac{2N_r}{NTU_x} + 1} \right] T_{xi}$$

Multiplying out the common denominator of all terms in brackets gives

$$\left[\frac{4N_x N_r C_r}{(hA)_x NTU_x} + \frac{2N_x C_r}{(hA)_x} + \frac{2N_r}{NTU_x} + 1 - 1 \right] T_{ro} = \left[\frac{4N_x N_r C_r}{(hA)_x NTU_x} + \frac{2N_x C_r}{(hA)_x} - \frac{2N_r}{NTU_x} - 1 + 1 \right] T_{ri} + \left[\frac{4N_r}{NTU_x} + 2 - 2 \right] T_{xi}$$

Multiplying through by $NTU_x/2N_r$ and adding and subtracting $2T_{ri}$ on the right gives

$$\left[\frac{2N_x C_r}{(hA)_x} + \frac{N_x C_r}{N_r C_{max}} + 1 \right] T_{ro} = \left[\frac{2N_x C_r}{(hA)_x} + \frac{N_x C_r}{N_r C_{max}} + 1 \right] T_{ri} + 2(T_{xi} - T_{ri})$$

Now solving for T_{ro} gives a form convenient for iterative calculations, thus,

$$T_{ro} = T_{ri} + 2 \left[1 + \frac{C_r N_x}{C_{max} N_r} + \frac{2N_x C_r}{(hA)_x} \right]^{-1} (T_{xi} - T_{ri}) \dots [20]$$

The term in brackets is constant for a given problem so that

$$T_{ro} = T_{ri} + K_2 (T_{xi} - T_{ri}) \dots \dots \dots [6b]$$

where K_2 may be put directly in terms of the dimensionless parameters giving

$$K_2 = 2 \left[1 + \frac{C_{\min}}{C_{\max}} \frac{C_r}{C_{\min}} \frac{N_r}{N_n} + \frac{2 N_r (hA)' \frac{C_r}{C_{\min}}}{NTU_0 [1 + (hA)']} \right]^{-1} \quad \dots \dots [8b]$$

For the element on the side of C_{\min} , Fig 2(b), a set of equations similar to equations [2] through [5] can be written. They are:

$$Q = C_{\min} (T_{no} - T_{ni}) (1/N_n) \quad \dots \dots [21]$$

$$Q = C_r (T_{ri} - T_{ro}) (1/N_r) \quad \dots \dots [22]$$

$$Q = (hA)_n \Delta T_{avg} (1/N_r 1/N_n) \quad \dots \dots [23]$$

$$\Delta T_{avg} = (1/2)(T_{ri} + T_{ro}) - (1/2)(T_{ni} + T_{no}) \quad \dots \dots [24]$$

Following the same procedure as before results in two equations similar to equations [13] and [14] as follows

$$T_{no} = \left[\frac{(hA)_n}{2N_r C_{\min} + (hA)_n} \right] (T_{ri} + T_{ro}) + \left[\frac{2N_r C_{\min} - (hA)_n}{2N_r C_{\min} + (hA)_n} \right] T_{ni} \quad \dots \dots [25]$$

and

$$T_{ro} = \left[\frac{(hA)_n}{2N_n C_r + (hA)_n} \right] (T_{ni} + T_{no}) + \left[\frac{2N_n C_r - (hA)_n}{2N_n C_r + (hA)_n} \right] T_{ri} \quad \dots \dots [26]$$

These may be solved as before for the outlet temperatures in terms of only the inlet temperatures giving

$$T_{no} = T_{ni} - 2 \left[1 + \frac{C_{\min}}{C_r} \frac{N_r}{N_n} + \frac{2N_r}{NTU_n} \right]^{-1} (T_{ni} - T_{ri}) \quad \dots \dots [27]$$

and

$$T_{ro} = T_{ri} + 2 \left[1 + \frac{C_r}{C_{\min}} \frac{N_n}{N_r} + \frac{2N_n C_r}{(hA)_n} \right]^{-1} (T_{ni} - T_{ri}) \dots [28]$$

The derivation of equations [27] and [28] is also readily apparent by comparison of equation [25] with [13] and [26] with [14]. The only difference is the substitution of the subscript "min" for "max" and "n" for "x" and then equation [27] follows from [19] and equation [28] follows from [20].

Equations [27] and [28] may be written as

$$T_{no} = T_{ni} + K_3 (T_{ri} - T_{ni}) \dots [7a]$$

and

$$T_{ro} = T_{ri} - K_4 (T_{ri} - T_{ni}) \dots [7b]$$

where

$$K_3 = 2 \left[1 + \frac{1}{\frac{C_r}{C_{\min}} \frac{N_r}{N_n}} + \frac{2N_r}{NTU_0 [1 + (hA)]} \right]^{-1} \dots [8c]$$

and

$$K_4 = 2 \left[1 + \frac{C_r}{C_{\min}} \frac{N_r}{N_n} + \frac{2N_n \frac{C_r}{C_{\min}}}{NTU_0 [1 + (hA)]} \right]^{-1} \dots [8d]$$

Equations [13], [14], [25] and [26] are repeated here for convenience. On the side of C_{\max} :

$$T_{x0} = \left[\frac{(hA)_x}{2N_r C_{\max} + (hA)_x} \right] (T_{ri} + T_{ro}) + \left[\frac{2N_r C_{\max} - (hA)_x}{2N_r C_{\max} + (hA)_x} \right] T_{xi} \dots [13]$$

$$T_{ro} = \left[\frac{(hA)_x}{2N_x C_r + (hA)_x} \right] (T_{xi} + T_{x0}) + \left[\frac{2N_x C_r - (hA)_x}{2N_x C_r + (hA)_x} \right] T_{ri} \dots [14]$$

and on the side of C_{min} :

$$T_{no} = \left[\frac{(hA)_n}{2N_r C_{min} + (hA)_n} \right] (T_{ri} + T_{ro}) + \left[\frac{2N_r C_{min} - (hA)_n}{2N_r C_{min} + (hA)_n} \right] T_{ni} \dots [25]$$

$$T_{ro} = \left[\frac{(hA)_n}{2N_n C_r + (hA)_n} \right] (T_{ni} + T_{no}) + \left[\frac{2N_n C_r - (hA)_n}{2N_n C_r + (hA)_n} \right] T_{ri} \dots [26]$$

It is obvious that the first term on the right of each of the foregoing equations is a positive quantity. It can next be observed that the coefficient of the second term of each equation must also be positive for if it were not the outlet temperature in question would grow hotter as its inlet temperature became colder, or vice versa, which would be contrary to the second law of thermodynamics.

This provides a convenient set of conditions to be met before the iterative scheme can be expected to converge. By using the relations of equations [15] through [18] these conditions may be stated as follows:

$$NTU_o \left[1 + \frac{1}{(hA)'} \right] \frac{C_{min}}{C_{max}} \leq 2N_r \dots [27]$$

$$\frac{NTU_o \left[1 + \frac{1}{(hA)'} \right]}{C_r/C_{min}} \leq 2N_r \dots [27]$$

$$NTU_o \left[1 + \frac{1}{(hA)'} \right] \leq 2N_r \dots [28]$$

$$\frac{NTU_o \left[1 + \frac{1}{(hA)'} \right]}{C_r/C_{min}} \leq 2N_n \dots [29]$$

In the ranges of parameters considered and by using an equal number of subdivisions of each stream it can be seen that condition [9] is the most severe and therefore it alone can be used as the condition for convergence.

10. Appendix 2 - Computer Program Details

The computer used for this work was the National Cash Register 102A general purpose digital computer and what follows assumes the reader has some familiarity with its operation.

The basic arrangement of the computer program was designed to accommodate any number of subdivisions of each of the three streams up to and including 64. It was found that, on the average, five iterations were required to meet the reversal condition and limit on the heat balance error with a judicious estimate of the initial matrix temperature distribution. With 16 subdivisions of each stream the time required for each iteration was four minutes. For different degrees of subdivision the time required for each iteration would vary, as would be expected, as the square of the factor by which the subdivision was changed, e.g. 32 subdivisions of each stream would require 16 minutes for each iteration. It was determined that 16 subdivisions of each stream could be expected to give accuracy to about five units in the fourth place of the effectiveness. To finish in a reasonable time and obtain the desired four place accuracy it was decided that all the results would be calculated using 16 subdivisions of each stream to be followed by sufficient check calculations with 32 subdivisions to permit extrapolation of the results to infinite subdivision.

The use of the various cells of the memory was as follows:

Cells	Use
0000-0477	Program
0500-0577	Initial T_r 's
0600-0777	Program
1000-1077	Current parameters and K's, and temporary storage.

1100-1177	Outlet temperatures on side of C_{\max}
1300-1377	Outlet temperatures on side of C_{\min}
1500-1577	Initial-Intermediate-Final T_r 's

The composite computer program flow diagram is shown in Fig. 14 and the contents of cells 0000-0477 and 0600-0777 are recorded in Table 10.

Box 10 and 11 of Fig. 14 accomplish the iteration process. This is done by the repetitive application of equations [6] to each element on the side of C_{\max} in box 10 and equations [7] to each element on the side of C_{\min} in box 11. Since the internal temperatures are of no concern in this problem it was only necessary to provide permanent storage for the fluid outlet temperatures of each column and the initial and final rotor temperatures of each row.

Box 12 determines if the reversal condition is met by taking the difference between the initial and final T_r of each row. A difference greater than the specified limit will cause the iteration cycle to start over. Also the largest difference is selected so that if the reversal condition is met a means of manual inspection is provided, if desired, by causing this difference to be printed out upon putting switch 2020 in the up position (box 17).

If the reversal condition is met then the heat balance error is computed in box 16 and if this is greater than the specified limit the iteration cycle is started over. The equation used to compute the heat balance error was arrived at as follows:

$$\begin{aligned}
 \text{error} &= \frac{Q_x - Q_n}{Q_x} = 1 - \frac{Q_n}{Q_x} \\
 &= 1 - \frac{C_{\min}(T_{n0} - T_{ni})}{C_{\max}(T_{xi} - T_{x0})}
 \end{aligned}$$

where T_{no} and T_{xo} are the average fluid outlet temperatures. Since T_{xi} was taken as unity and T_{ni} as zero then the heat balance error reduces to

$$\text{error} = 1 - \frac{\frac{C_{\min}}{C_{\max}} (\bar{T}_{no})_{\text{avg}}}{1 - (\bar{T}_{xo})_{\text{avg}}} \quad [30]$$

The average fluid outlet temperature on each side was computed by summing the individual outlet temperatures of each column and dividing by the number of subdivisions.

Another means of optional manual inspection was provided in box 19. If desired, the heat balance error would be converted from octal to decimal and printed out by putting switch 2040 in the up position.

By definition, the effectiveness may be computed by either of the following equations:

$$E = \frac{C_h (\bar{T}_{hi} - \bar{T}_{ho})}{C_{\min} (\bar{T}_{hi} - \bar{T}_{ci})} \quad [31a]$$

$$E = \frac{C_c (\bar{T}_{co} - \bar{T}_{ci})}{C_{\min} (\bar{T}_{hi} - \bar{T}_{ci})} \quad [31b]$$

where \bar{T}_{ho} and \bar{T}_{co} are the average fluid outlet temperatures on the hot and cold sides, respectively. From the conditions assumed for calculation purposes the following may be observed:

$$T_{ci} = T_{ni} = 0$$

$$T_{hi} = T_{xi} = 1$$

$$C_c = C_{\min}, \text{ and,}$$

$$T_{co} = T_{no}$$

and therefore the expression for effectiveness from equation [31b] reduces to

$$E = (\bar{T}_{no})_{\text{avg}} \quad [32]$$

$(T_{no})_{avg}$ will have already been computed in the process of calculating the heat balance error and therefore in box 21 it is only necessary to convert this to decimal and print it out.

The final T_r 's punched on cards in box 22 were used as estimates for the initial T_r 's in subsequent calculations. This was usually done by using the final T_r 's obtained at a given C_{min}/C_{max} as the initial T_r 's for the next lower value of C_{min}/C_{max} , at corresponding values of the other parameters.

The remainder of the program consists of a provision for interrupting the calculations at logical stopping places and a system of inserting new values of parameters after each solution. These are self-evident in Fig. 14. Parameters are entered as floating point numbers and the arithmetic involved in calculating the K's is done in floating point. The K's, scaled down by a factor of two, are then converted to single precision and the subsequent iteration is accomplished in fixed point, single precision.

The basic computer program which is recorded in Table 10 is set up to use 16 subdivisions of each stream. To modify it for 8 or 32 subdivisions the following cells should be changed as indicated:

Cell	8 Subdivisions	32 Subdivisions
0307	21050021000010	21050021000040
0403	5	7
0406	35051021001510	35054021001540
0410	36200015102001	36200015402001
0411	35200021001110	35200021001140
0424	36150720002001	36153720002001
0425	36150720031507	36153720031537

0430	35200021001310	35200021001340
0433	2ff0003	2ff0005
0436	30131004332007	30134004332007
0437	30111004332007	30114004332007
0443	35151021000510	35154021000540
0447	36051015102000	36054015402000
0456	04300030000510	04300030000540
0462	05300030001510	05300030001540

In its present form the program by-passes box 25 and goes directly to box 27. This causes all calculations to be made with $(hA)' = 1$. To get results with $(hA)' = 1, 1/2, \text{ and } 1/4$ enter the following changes:

0007	21100604020002
0367	33100606760371
0370	34300021000373
0371	36100604001006
0372	34300021000006

In its present form the program produces results with $C_r/C_{\min} = 1, 1.25, 1.5, 2, 3, \text{ and } 5$. To eliminate $C_r/C_{\min} = 5$ enter the following changes:

0375	34067210020600
0376	34067310030600

Space is provided in the iteration subroutines for entering appropriate commands to get any desired temperatures printed out before they are lost. These may be printed in decimal simply by placing in cell 2000 and unconditional transfer to the decimal conversion and print subroutine at cell 0700.

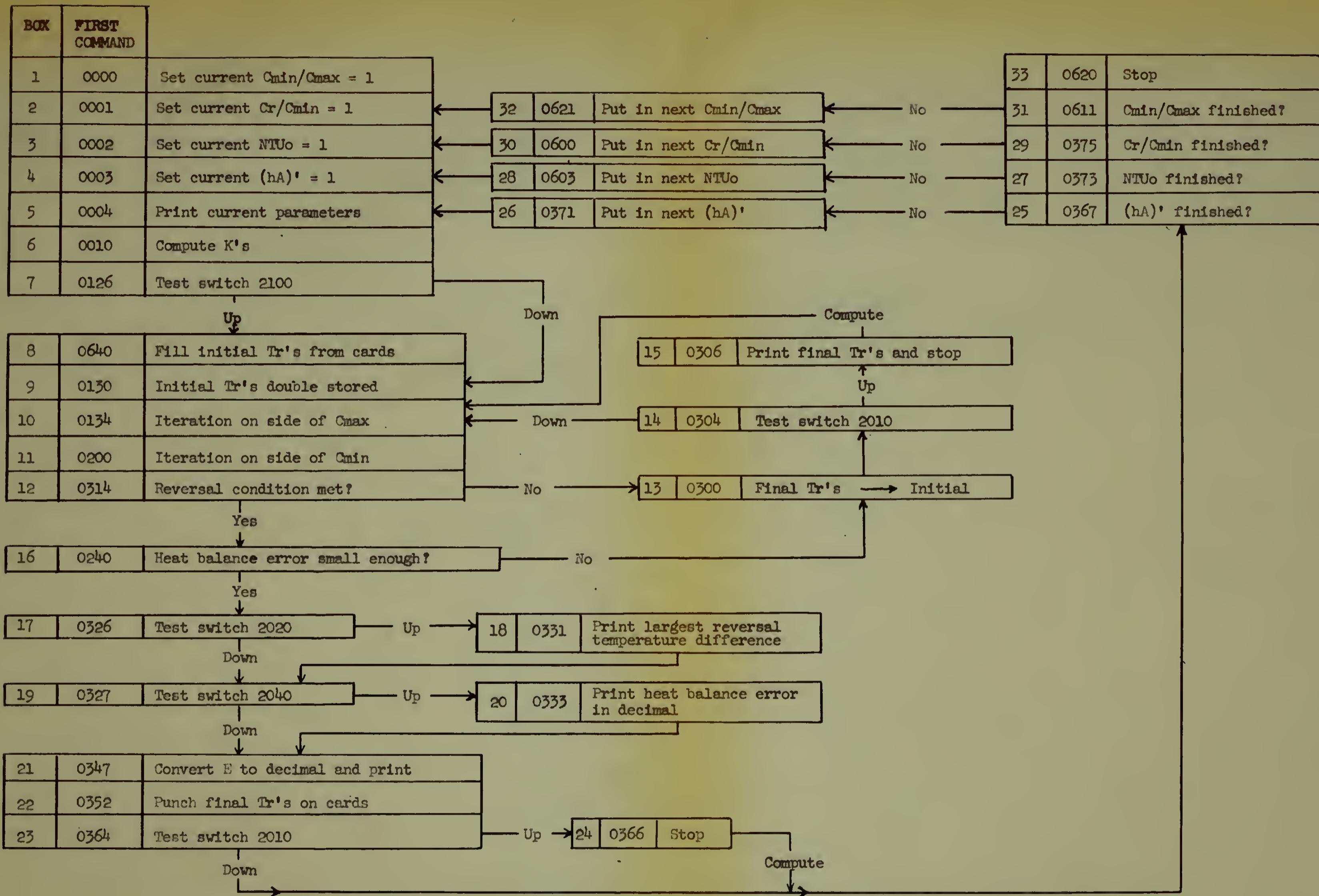


Fig. 14 COMPUTER PROGRAM FLOW DIAGRAM

TABLE 10 COMPUTER PROGRAM COMMANDS

0000 31040004011000	0032 34300021000760
0001 31040004011002	0033 31200420051046
0002 31040004011004	0034 31104210432000
0003 31040004011006	0035 34300021000725
0004 21100004020002	0036 31200420051050
0005 21100204020002	0037 31100010012002
0006 21100404020002	0040 34300021000760
0007 34300021000010	0041 31200420051052
0010 31040004012000	0042 31100210032000
0011 31100610072002	0043 34300021000725
0012 34300021000733	0044 31200420051054
0013 31200420052000	0045 31200420052002
0014 31100410052002	0046 31040004012000
0015 34300021000725	0047 34300021000760
0016 31200420052002	0050 31200420051056
0017 31040304012000	0051 31105210532000
0020 34300021000760	0052 31105610572002
0021 31200420051040	0053 34300021000733
0022 31200420052000	0054 31200420051060
0023 31100610072002	0055 31200420052002
0024 34300021000725	0056 31040004012000
0025 31200420051042	0057 34300021000733
0026 31100210032002	0060 31200420052002
0027 34300021000725	0061 34300021000760
0030 31200420051044	0062 31200420051010
0031 31040004012000	0063 31105010512000

TABLE 10 CONTINUED

0064	31105410552002	0116	30101110101011
0065	34300021000733	0117	30101310121013
0066	31200420052002	0120	30101510141015
0067	31040004012000	0121	30101710161017
0070	34300021000733	0122	33210010110010
0071	31200420052002	0123	33210010130010
0072	34300021000760	0124	33210010150010
0073	31200420051012	0125	33210010170010
0074	31104010412000	0126	17210030000640
0075	31104610472002	0127	34300021000130
0076	34300021000733	0130	35040421000131
0077	31200420052002	0131	35052021001520
0100	31040004012000	0132	35013104050131
0101	34300021000733	0133	34040601310131
0102	31200420052002	0134	35041221000157
0103	34300021000760	0135	35041321000141
0104	31200420051014	0136	35041421000146
0105	31104410452000	0137	34300021000140
0106	31100210032002	0140	35040121002000
0107	34300021000733	0141	36200015202001
0110	31200420052002	0142	30200104002001
0111	31040004012000	0143	25200110112002
0112	34300021000733	0144	25200110132003
0113	31200420052002	0145	36200020022000
0114	34300021000760	0146	35152020031520
0115	31200420051016	0147	34300021000154

TABLE 10 CONTINUED

0150	0000000000000000	0202	35042521000211
0151	0000000000000000	0203	35210021002000
0152	0000000000000000	0204	36147720002001
0153	0000000000000000	0205	30200104002001
0154	35014104070141	0206	25200110152002
0155	35014604050146	0207	25200110172003
0156	34041001410141	0210	35200020022000
0157	35200021001120	0211	36147720031477
0160	34300021000170	0212	34300021000216
0161	0000000000000000	0213	0000000000000000
0162	0000000000000000	0214	0000000000000000
0163	0000000000000000	0215	0000000000000000
0164	0000000000000000	0216	36020404150204
0165	0000000000000000	0217	36021104050211
0166	0000000000000000	0220	34020404260204
0167	0000000000000000	0221	35200021001320
0170	35015704000157	0222	34300021000231
0171	34041101570135	0223	0000000000000000
0172	34300021000200	0224	0000000000000000
0173	0000000000000000	0225	0000000000000000
0174	0000000000000000	0226	0000000000000000
0175	0000000000000000	0227	0000000000000000
0176	0000000000000000	0230	0000000000000000
0177	0000000000000000	0231	35022104000221
0200	35042721000221	0232	34043002210201
0201	35042421000204	0233	34300021000314

TABLE 10 CONTINUED

0234 0000000000000000	0266 30200520042005
0235 0000000000000000	0267 36040120052005
0236 0000000000000000	0270 30200504002005
0237 0000000000000000	0271 34200504400300
0240 35043421000243	0272 34300021000326
0241 35043521000255	0273 0000000000000000
0242 35210021002000	0274 0000000000000000
0243 30132004332007	0275 0000000000000000
0244 35200020072000	0276 0000000000000000
0245 35024304150243	0277 0000000000000000
0246 34043602430243	0300 35044221000300
0247 30200004001040	0301 35150021000500
0250 31210020002000	0302 35030104050301
0251 31100010012002	0303 34044303010301
0252 34300021000725	0304 17201030000306
0253 31200420052000	0305 34300021000134
0254 35210021002003	0306 21044404450001
0255 30112004332007	0307 21050021000020
0256 35200320072003	0310 2200000000000000
0257 35025504150255	0311 34300021000134
0260 34043702550255	0312 0000000000000000
0261 36040120032003	0313 0000000000000000
0262 31210020032002	0314 35210021001050
0263 35200204002002	0315 35044621000316
0264 34300021000265	0316 36052015202000
0265 34300021000760	0317 34200010500324

TABLE 10 CONTINUED

0320	35031604500316	0352	35045721000353
0321	34044703160316	0353	05300030001520
0322	34105004410300	0354	13046004550000
0323	34300021000240	0355	30300004611077
0324	35200021001050	0356	13046004550004
0325	34300021000320	0357	30300004611077
0326	17202030000331	0360	35035304530353
0327	17204030000333	0361	34046203530353
0330	34300021000340	0362	34300021000363
0331	21105021000001	0363	34300021000364
0332	34300021000327	0364	17201030000366
0333	35200521002000	0365	34300021000367
0334	34300021000700	0366	2200000000000000
0335	34300021000340	0367	34300021000373
0336	0000000000000000	0370	0000000000000000
0337	0000000000000000	0371	0000000000000000
0340	34300021000347	0372	0000000000000000
0341	0000000000000000	0373	34065210040603
0342	0000000000000000	0374	34065310050603
0343	0000000000000000	0375	34067410020600
0344	0000000000000000	0376	34067510030600
0345	0000000000000000	0377	34300021000611
0346	0000000000000000	0400	0000000000000000
0347	35104021002000	0401	0040000000000000
0350	34300021000700	0402	0200000000000000
0351	21044404450001	0403	0000000000000006

TABLE 10 CONTINUED

0404 35050021001500	0436 30132004332007
0405 00000100000001	0437 30112004332007
0406 35052021001520	0440 00000100000000
0407 00000000010000	0441 00000004000000
0410 36200015202001	0442 35150021000500
0411 35200021001120	0443 35152021000520
0412 35200021001100	0444 00341212341212
0413 36200015002001	0445 10000000000000
0414 35150020031500	0446 36050015002000
0415 00000100000000	0447 36052015202000
0416 30050004002000	0450 00000100010000
0417 30052004002000	0451 04300030000500
0420 30152004002000	0452 15555555555556
0421 30150004002000	0453 00000000000010
0422 30110004002000	0454 00000000000500
0423 30112004002000	0455 11111111111111
0424 36151720002001	0456 04300030000520
0425 36151720031517	0457 05300030001500
0426 36147720002001	0460 26666666666666
0427 35200021001300	0461 00000000001000
0430 35200021001320	0462 05300030001520
0431 30130004002000	0463 00000200020000
0432 30132004002000	0464 31066406651002
0433 02000000000004	0465 31210006551000
0434 30130004332007	0466 00000000000000
0435 30110004332007	0467 00000000000000

TABLE 10 CONTINUED

0470 0000000000000000	0622 35062104070621
0471 0000000000000000	0623 34300021000001
0472 0000000000000000	0624 0000000000000000
0473 0000000000000000	0625 0000000000000000
0474 0000000000000000	0626 0000000000000000
0475 0000000000000000	0627 0000000000000000
0476 0000000000000000	0630 0000000000000000
0477 0000000000000000	0631 0000000000000000
0600 31066406651002	0632 0000000000000000
0601 35060004630600	0633 0000000000000000
0602 34300021000002	0634 0000000000000000
0603 31040004012000	0635 0000000000000000
0604 31100410052002	0636 0000000000000000
0605 34300021000733	0637 0000000000000000
0606 31200420051004	0640 35045121000646
0607 31040004011006	0641 05300030002100
0610 34300021000006	0642 06045204550000
0611 35046421000600	0643 30300004541077
0612 31100010012000	0644 06045204550004
0613 35210021002002	0645 30300004541077
0614 36210006632003	0646 04300030000520
0615 34300021000733	0647 35064604530646
0616 34200521000621	0650 34045606460641
0617 35046521000621	0651 34300021000130
0620 2200000000000000	0652 00000000000004
0621 31210006551000	0653 0050000000000000

TABLE 10 CONTINUED

0654 000000000000001	0706 32200220012000
0655 00746314631463	0707 32210020012002
0656 00714631463146	0710 27200120052001
0657 00631463146315	0711 34200121000705
0660 00546314631463	0712 21200007200001
0661 004000000000000	0713 34300021000351
0662 00231463146315	0714 34300021000000
0663 00063146314632	0715 020000000000004
0664 000000000000001	0716 02000000000030
0665 005000000000000	0717 005000000000000
0666 000000000000001	0720 010000000000000
0667 006000000000000	0721 007400000000000
0670 000000000000002	0722 00463511721510
0671 004000000000000	0723 020000000000001
0672 000000000000002	0724 000000000000001
0673 006000000000000	0725 30300007162004
0674 000000000000003	0726 35071420040732
0675 005000000000000	0727 35200020022004
0676 020000000000001	0730 25200120032005
0677 004000000000000	0731 31200420052004
0700 27300007162001	0732 34300021000253
0701 35071420010713	0733 30300007162004
0702 35200021000722	0734 35071420040745
0703 05300030000714	0735 33200020020746
0704 32200207002000	0736 36200020022004
0705 25200220072002	0737 30200120042005

- TABLE 10 CONTINUED

0740 35200321002006	0772 35200407242004
0741 35200221002004	0773 34300021000765
0742 35200520062005	0774 0000000000000000
0743 37200521000753	0775 0000000000000000
0744 31200420052004	0776 0000000000000000
0745 34300021000113	0777 0000000000000000
0746 36200220002004	
0747 30200320042005	
0750 35200121002006	
0751 35200021002004	
0752 34300021000742	
0753 27200507232005	
0754 30200507232005	
0755 27200507242005	
0756 35200407242004	
0757 34300021000744	
0760 30300007162004	
0761 35071420040766	
0762 36200020022004	
0763 23200120032005	
0764 37200521000767	
0765 31200420052004	
0766 34300021000266	
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JA 28 64	1 3 1 0 9

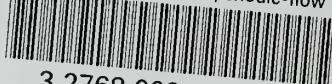
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a periodic-flow heat ex-
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